Exam 3 Math 2306 sec. 54

Fall 2021

Name: (4 pts) Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	
Total (+4)	

INSTRUCTIONS: There are 6 problems worth 16 points each. You may use one sheet (8.5" × 11") of your own prepared notes/formulas.
No use of a calculator, text book, smart device, or other resource is permitted. Illicit use of any additional resource will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.

Show all of your work on the paper provided to receive full credit.

1. Evaluate each Laplace transform.

(a) $\mathscr{L}\left\{e^{3t}+3\cos(7t)\right\} = \mathscr{L}\{e^{3t}\} + 3\mathscr{L}\{\cos(7t)\} = \frac{1}{s-3} + \frac{3s}{s^2+49}$

(b) $\mathscr{L}\left\{(2+t^2)^2\right\} = 4\mathscr{L}\{1\} + 4\mathscr{L}\{t^2\} + \mathscr{L}\{t^4\} = \frac{4}{s} + 4\frac{2!}{s^3} + \frac{4!}{s^5}$

$(2+t^2)^2 = 4+4t^2+t^4$

(c) $\mathscr{L}\left\{2\sin(3t)\cos(3t)\right\} = \mathscr{L}\{\sin(6t)\} = \frac{6}{s^2+36}$

$2\sin(3t)\cos(3t) = \sin(6t)$

2. Find the steady state **charge**, q_p, on the capacitor in the LRC series circuit described by the given equation.

$$q'' + 2q' + 5q = 10\sin t$$

(**Note:** The transient charge is $q_c = c_1 e^{-t}\cos(2t) + c_2 e^{-t}\sin(2t)$.)

Let

This has no like terms in common w/ q_c

5 $\quad q_p = A\sin t + \cos t$

2 $\quad q_p' = A\cos t - B\sin t$

1 $\quad q_p'' = -A\sin t - B\cos t$

$$q_p'' + 2q_p' + 5q_p =$$

$$(-A - 2B + 5A)\sin t + (-B + 2A + 5B)\cos t = 10\sin t$$

$$4A - 2B = 10$$

$$2A + 4B = 0 \Rightarrow A = -2B$$

$$-8B - 2B = 10 \Rightarrow B = -1 \quad A = 2$$

The steady state charge

$$\boxed{q_p = 2\sin t - \cos t}$$

3. Evaluate each inverse Laplace transform.

(a) $\mathscr{L}^{-1}\left\{\dfrac{3s}{s^2+16}\right\} = 3\,\mathscr{L}^{-1}\left\{\dfrac{s}{s^2+16}\right\} = 3\cos(4t)$

(b) $\mathscr{L}^{-1}\left\{\dfrac{1}{s^9}\right\} = \dfrac{1}{8!}\,\mathscr{L}^{-1}\left\{\dfrac{8!}{s^9}\right\} = \dfrac{1}{8!}\,t^8$

(c) $\mathscr{L}^{-1}\left\{\dfrac{s}{(s-1)(s+3)}\right\} = \dfrac{1}{4}\,\mathscr{L}^{-1}\left\{\dfrac{1}{s-1}\right\} + \dfrac{3}{4}\,\mathscr{L}^{-1}\left\{\dfrac{1}{s+3}\right\}$

$= \dfrac{1}{4}e^{t} + \dfrac{3}{4}e^{-3t}$

$\dfrac{s}{(s-1)(s+3)} = \dfrac{A}{s-1} + \dfrac{B}{s+3}$

$s = A(s+3) + B(s-1)$

Set $s=1$ $\quad 1 = 4A \quad A = \frac{1}{4}$

$s=-3 \quad -3 = -4B \quad B = 3/4$

4. Suppose that f is defined on $[0,\infty)$ and $\mathscr{L}\{f(t)\} = \dfrac{1}{\sqrt{s^2+9}}$. Evaluate

(a) $\mathscr{L}\{e^{2t}f(t)\} = \dfrac{1}{\sqrt{(s-2)^2+9}}$

(b) $\mathscr{L}\{f(t-\pi)\mathscr{U}(t-\pi)\} = \dfrac{e^{-\pi s}}{\sqrt{s^2+9}}$

$F(s) = \dfrac{1}{\sqrt{s^2+9}}$, $\quad F(s-2)$, $\quad e^{-\pi s}F(s)$

5. A 3 kg mass is attached to a spring with spring constant 27 N/m.

(a) If there is no damper, and a driving force $f(t) = \sin(\gamma t)$ is applied, what value of γ will result in pure resonance?

Pure resonance $\Rightarrow$ $\gamma = \omega$. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{27}{3}} = 3$

$\boxed{\text{The resonance frequency } \gamma = 3.}$

(b) If there is no driver, but a dashpot is added to induce damping of β N per m/sec of velocity, what value of β will result in critical damping?

$3x'' + \beta x' + 27x = 0$ critical damping $\Rightarrow$ $\beta^2 - 4mk = 0$

$\beta = \sqrt{4mk} = \sqrt{4 \cdot 3 \cdot 27} = 2 \cdot 3^2 = 18$

$\boxed{\beta = 18}$

6. Evaluate each Inverse Laplace transform.

(a) $\mathscr{L}^{-1}\left\{\frac{e^{-3s}}{s^9}\right\} = \frac{1}{8!}(t-3)^8 u(t-3)$

for $\frac{1}{s^9}$ $f(t) = \frac{1}{8!} t^8$

(b) $\mathscr{L}^{-1}\left\{\frac{s}{s^2 - 4s + 5}\right\} = \mathscr{L}^{-1}\left\{\frac{s-2}{(s-2)^2+1}\right\} + 2\,\mathscr{L}^{-1}\left\{\frac{1}{(s-2)^2+1}\right\}$

$= e^{2t}\cos t + 2e^{2t}\sin t$

$s^2 - 4s + 5 = (s-2)^2 + 1$

$\frac{s}{(s-2)^2+1} = \frac{s-2+2}{(s-2)^2+1} = \frac{s-2}{(s-2)^2+1} + \frac{2}{(s-2)^2+1}$