Home work 3: Due Thurs. Feb. 11, 2016 Math 2335 Spring 2016

Name: ____

(1) Let $f(x) = (x-1)(x-2)(x-3)$. Note that $f(1) = 0$. Let $x_T = 1$, and $x_A = 1 + 10^{-4}$. Use the formula for propagated error (e.g. (2.43) from page 60 in Atkinson and Han) to show that the propagated error $E = f(x_T) - f(x_A)$ is about double the error $\mathrm{Err}(x_A)$.[1] (Hint: It is best to use the product rule when computing $f'(x)$ rather than expanding f.)

[1]Two point bonus: Can you extend this to determine how the propogated error would be related to $\mathrm{Err}(x_A)$ if

$$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)(x-8)?$$

(2) Let $g(x) = e^{-x}$ and $h(x) = \ln(x+1)$. Demonstrate graphically that there is a solution to the equation $g(x) = h(x)$. Use the bisection method with a hand calculator or a computer to find the root accurate to within $\epsilon = 0.01$. Produce a table of your iterates with the following columns: n, a_n, b_n, c_n, $b_n - c_n$. (Hand written is fine.) For example:

n	a_n	b_n	c_n	$b_n - c_n$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(3) Let α be the unique positive root of $f(x)$. Find an interval $[a, b]$ containing α for which the bisection method will converge to α. Then, estimate the number of iterates needed to find α within an accuracy or $\epsilon = 10^{-9}$. Note: Your answer should be justified and will depend on the choice of $[a, b]$.

(a) $f(x) = 31x^3 - x^2 + 27x - 2125$

(b) $f(x) = e^x - x - 2$