Home work 9: Due Thurs. April 28, 2016 Math 2335 Spring 2016

Name: _____

(1) Consider the numerical approximation to $f'(x)$ given by

$$f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}.$$

Approximate $f'(x)$ for $f(x) = \ln x$ at $x = 1$ and try to determine the order[†] of the approximation. To do this, evaluate the approximation for $h_1 = 0.1$, $h_2 = 0.05$, $h_3 = 0.025$, $h_4 = 0.0125$, and $h_5 = 0.00625$ using 10 digits to the right of the decimal point. Compute the error E_i, $i = 1..5$ obtained using each h value. Use the ratios E_i/E_{i+1} to draw your conclusion. Fill in the following table:

i	h_i	$\frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$	Error E_i	E_i/E_{i+1}
1	0.1			
2	0.05			
3	0.025			
4	0.0125			
5	0.00625			

[†]If the error is proportional to h^p, then the order of the approximation is p.

(2) Use the method of undertermined coefficients to find a numerical differentiation formula for $f'(x)$ of the form

$$f'(x) \approx Af(x+2h) + Bf(x+h) + Cf(x).$$