January 19 Math 3260 sec. 52 Spring 2024

Section 1.3: Vector Equations

Definition: Vector

A matrix that consists of one column is called a **column vector** or simply a **vector**.

When we give a vector a name (i.e. use a variable to denote a vector), the convention

- in typesetting is to use bold face

$$\mathbf{u} \quad \text{and} \quad \mathbf{x}$$

- in handwriting is to place a little arrow over the variable

$$\vec{u} \quad \text{and} \quad \vec{x}$$

Set of Real Ordered Pairs: $\mathbb{R}^2$

The set of vectors of the form $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with x_1 and x_2 any real numbers is denoted by

$$\mathbb{R}^2$$

(read ”R two”). It’s the set of all real ordered pairs.

Geometry

Each vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ corresponds to a point in the Cartesian plane. We can equate them with ordered pairs written in the traditional format

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = (x_1, x_2).$$

This is **not to be confused with a row matrix.**

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \neq [x_1 \ \ x_2]$$

We can identify vectors with points or with directed line segments emanating from the origin (little arrows).

Geometry

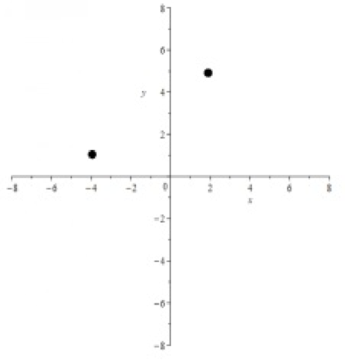

Figure: Vectors characterized as points, and vectors characterized as directed line segments.

$$\begin{bmatrix} -4 \\ 1 \end{bmatrix} = (-4, 1), \quad \begin{bmatrix} 2 \\ 5 \end{bmatrix} = (2, 5)$$

Vector Equality

Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, and c be a scalar*.

Vector Equivalence: Equality of vectors is defined by

$$\mathbf{u} = \mathbf{v} \quad \text{if and only if} \quad u_1 = v_1 \quad \text{and} \quad u_2 = v_2.$$

*A **scalar** is an element of the set from which u_1 and u_2 come. For our purposes, a scalar is a *real* number.

Algebraic Operations

Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, and c be a scalar.

Scalar Multiplication: The scalar multiple of **u**

$$c\mathbf{u} = \begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}.$$

Vector Addition: The sum of vectors **u** and **v**

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}$$

Examples

Let $\mathbf{u} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -3 \\ \frac{3}{2} \end{bmatrix}$

Evaluate

(a) $-2\mathbf{u}$

By the definition

$$-2\vec{u} = -2 \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 4 \end{bmatrix}$$

Examples

Let $\mathbf{u} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -3 \\ \frac{3}{2} \end{bmatrix}$

Evaluate

(b) $-2\mathbf{u}+3\mathbf{v}$

We know that $-2\vec{u} = \begin{bmatrix} -8 \\ 4 \end{bmatrix}$

Also, $3\vec{v} = 3\begin{bmatrix} -1 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 \\ 21 \end{bmatrix}$.

So $-2\vec{u} + 3\vec{v} = \begin{bmatrix} -8 \\ 4 \end{bmatrix} + \begin{bmatrix} -3 \\ 21 \end{bmatrix} = \begin{bmatrix} -11 \\ 25 \end{bmatrix}$

Examples

$$\text{Let}\quad \mathbf{u} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}, \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} -3 \\ \frac{3}{2} \end{bmatrix}$$

(c) Is it true that $\mathbf{w} = -\frac{3}{4}\mathbf{u}$?

well, $-\frac{3}{4}\vec{u} = \begin{bmatrix} -3 \\ \frac{3}{2} \end{bmatrix}$, so yes, $\vec{w} = -\frac{3}{4}\vec{u}$.