Math 2306 Lecture 19

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Outline

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Method of Undetermined Coefficients

Two methods for finding a particular solution y_P

- Method of undetermined coefficients: $ay'' + by' + cy = g(t)$
- Method of variation of parameters [Section 2.5.3]

Method of Undetermined Coefficients

Two methods for finding a particular solution y_P

- Method of undetermined coefficients: $ay'' + by' + cy = g(t)$
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EXAMPLE 2.16 Solve the differential equation

$$y'' + 3y' + 2y = 5e^{2t}.$$

Example 2.16 Continued

Example 2.18

Find the general solution for $y'' + 3y' + 2y = 5e^{-2t}$.

Similar Table to the One on Page 156 of Textbook

TABLE 3.1

The right-hand column gives the proper form to assume for a particular solution of $ay'' + by' + cy = g(t)$. In the right-hand column, choose r to be the smallest nonnegative integer such that no term in the assumed form is a solution of the homogeneous equation $ay'' + by' + cy = 0$. The value of r will be 0, 1, or 2.

Form of $g(t)$	Form to Assume for a Particular Solution $y_P(t)$
$a_n t^n + \cdots + a_1 t + a_0$	$t^r[A_n t^n + \cdots + A_1 t + A_0]$
$[a_n t^n + \cdots + a_1 t + a_0]e^{\alpha t}$	$t^r[A_n t^n + \cdots + A_1 t + A_0]e^{\alpha t}$
$[a_n t^n + \cdots + a_1 t + a_0]\sin \beta t$ or $[a_n t^n + \cdots + a_1 t + a_0]\cos \beta t$	$t^r[(A_n t^n + \cdots + A_1 t + A_0)\sin \beta t + (B_n t^n + \cdots + B_1 t + B_0)\cos \beta t]$
$e^{\alpha t}\sin \beta t$ or $e^{\alpha t}\cos \beta t$	$t^r[Ae^{\alpha t}\sin \beta t + Be^{\alpha t}\cos \beta t]$
$e^{\alpha t}[a_n t^n + \cdots + a_0]\sin \beta t$ or $e^{\alpha t}[a_n t^n + \cdots + a_0]\cos \beta t$	$t^r[(A_n t^n + \cdots + A_0)e^{\alpha t}\sin \beta t + (B_n t^n + \cdots + B_0)e^{\alpha t}\cos \beta t]$

<u>Caution:</u> This method has problems when proposed y_p contains elements of y_c.

Example 2.19

Find the correct form for $y_p(t)$ when

$$y'' + 4y = 2t^2 + 5\sin(2t) + e^{3t}.$$

Example 2.20

Find the general solution of $y'' - y' - 2y = 3t^3$.

Summary

Today we learned

- method of undetermined coefficients

next time, we will learn

- method of variation of parameters