Math 2306 Lecture 19

Dr. Lihong Zhao

lzhao12@kennesaw.edu

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Worksheet 02 Solutions (due by 11:59pm on Oct 2, 2025)

1. Calculate $\int xe^{x^2}\,\mathrm{d}x$ using the method integration by parts.

   Solution: Let $u = x^2$, then $\mathrm{d}u = 2x\,\mathrm{d}x$ and $x\,\mathrm{d}x = \frac{1}{2}\,\mathrm{d}u$. Thus

$$\int xe^{x^2}\,\mathrm{d}x = \frac{1}{2}\int e^u\,\mathrm{d}u = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2} + c$$

2. Calculate $\int \left(1 - \frac{1}{\omega}\right)\cos\left(\omega - \ln\omega\right)\mathrm{d}x$ using the method integration by substitution.

   Solution: Let $u = \omega - \ln\omega$, then $\mathrm{d}u = \left(1 - \frac{1}{\omega}\right)\mathrm{d}\omega$. Therefore,

$$\begin{aligned}\int \left(1 - \frac{1}{\omega}\right)\cos\left(\omega - \ln\omega\right)\mathrm{d}x &= \int \cos u\,\mathrm{d}u \\ &= \sin u + c \\ &= \sin(\omega - \ln\omega) + c\end{aligned}$$

Outline

1. Section 2.5 Nonhomogeneous Equations and Undetermined Coefficients
   - Method of Undetermined Coefficients

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Two methods for finding a particular solution y_P

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$$y_p' = 2Ae^{2t}; \quad y_p'' = 4Ae^{2t}.$$

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Plug into the nonhomogeneous differential equation

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we have

$$4Ae^{2t} + 3\left(2Ae^{2t}\right) + 2\left(Ae^{2t}\right) = 5e^{2t} \Rightarrow A = \frac{5}{12}$$

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$$y = y_c + y_p = c_1 e^{-t} + c_2 e^{-2t} + \frac{5}{12}e^{2t}.$$

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$$4Ae^{-2t} - 6Ae^{-2t} + 2Ae^{-2t} = e^{-2t}(4A - 6A + 2A) = 0.$$

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Plugging into the ODE, we have

$$\begin{aligned} 5e^{-2t} &= 4Ae^{-2t}(t-1) + 3Ae^{-2t}(1-2t) + 2Ate^{-2t} \\ &= Ae^{-2t}(4t - 4 + 3 - 6t + 2t) \\ &= -Ae^{-2t} \\ \Rightarrow \qquad & A = -5 \end{aligned}$$

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The general solution is $y = c_1 e^{-t} + c_2 e^{-2t} - 5te^{-2t}$.

Similar Table to the One on Page 156 of Textbook

TABLE 3.1

The right-hand column gives the proper form to assume for a particular solution of $ay'' + by' + cy = g(t)$. In the right-hand column, choose r to be the smallest nonnegative integer such that no term in the assumed form is a solution of the homogeneous equation $ay'' + by' + cy = 0$. The value of r will be 0, 1, or 2.

Form of $g(t)$	Form to Assume for a Particular Solution $y_P(t)$
$a_n t^n + \cdots + a_1 t + a_0$	$t^r[A_n t^n + \cdots + A_1 t + A_0]$
$[a_n t^n + \cdots + a_1 t + a_0]e^{\alpha t}$	$t^r[A_n t^n + \cdots + A_1 t + A_0]e^{\alpha t}$
$[a_n t^n + \cdots + a_1 t + a_0]\sin \beta t$ or $[a_n t^n + \cdots + a_1 t + a_0]\cos \beta t$	$t^r[(A_n t^n + \cdots + A_1 t + A_0)\sin \beta t + (B_n t^n + \cdots + B_1 t + B_0)\cos \beta t]$
$e^{\alpha t}\sin \beta t$ or $e^{\alpha t}\cos \beta t$	$t^r[Ae^{\alpha t}\sin \beta t + Be^{\alpha t}\cos \beta t]$
$e^{\alpha t}[a_n t^n + \cdots + a_0]\sin \beta t$ or $e^{\alpha t}[a_n t^n + \cdots + a_0]\cos \beta t$	$t^r[(A_n t^n + \cdots + A_0)e^{\alpha t}\sin \beta t + (B_n t^n + \cdots + B_0)e^{\alpha t}\cos \beta t]$

Caution: This method has problems when proposed y_p contains elements of y_c.

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Solution: From Theorem 2.6, we can break down the particular solution into parts. Let $y'' + 4y = g_i(t)$ with $i = 1, 2, 3$.

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<u>*Solution*</u>: From Theorem 2.6, we can break down the particular solution into parts. Let $y'' + 4y = g_i(t)$ with $i = 1, 2, 3$.

- $g_1(t) = 2t^2$, try $y_{p1}(t) = At^2 + Bt + C$.

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- $g_2(t) = 5\sin(2t)$, try $y_{p2}(t) = Dt\cos(2t) + Et\sin(2t)$ since the sine and cosine appear in the complementary solution.

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Then the particular solution is going to be

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Observe that the characteristic equation is $r^2 + 4 = 0$, and the roots are $r_{1,2} = \pm 2i$. Therefore the complementary solution is $y_c(t) = c_1\cos(2t) + c_2\sin(2t)$.

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$$\begin{aligned} 3t^3 &= (6At + 2B) - (3At^2 + 2Bt + C) - 2\left(At^3 + Bt^2 + Ct + D\right) \\ &= -2At^3 - (3A + 2B)t^2 + (6A - 2B - 2C)t + (2B - C - 2D) \end{aligned}$$

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Matching powers of t, we have a system of equations to solve:

$$\begin{cases} -2A = 3 \\ -3A - 2B = 0 \\ 6A - 2B - 2C = 0 \\ 2B - C - 2D = 0 \end{cases} \Rightarrow \begin{cases} A = -\frac{3}{2} \\ B = \frac{9}{4} \\ C = -\frac{27}{4} \\ D = \frac{45}{8} \end{cases}$$

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Thus $y_p = -\frac{3}{2}t^3 + \frac{9}{4}t^2 - \frac{27}{4}t + \frac{45}{8}$. The general solution is

$$y(t) = c_1 e^{-t} + c_2 e^{2t} - \frac{3}{2}t^3 + \frac{9}{4}t^2 - \frac{27}{4}t + \frac{45}{8}.$$

Worksheet 02 Solution Continued

Exercise 3. Calculate $\int \frac{x^2+3x+5}{x+1}\,\mathrm{d}x$ using the method of partial fraction decomposition.

Solution: First, let us do the long division

$$\begin{array}{r|l} & x+2 \\ \hline x+1) & x^2+3x+5 \\ & -x^2-x \\ \hline & 2x+5 \\ & -2x-2 \\ \hline & 3 \end{array}$$

Thus

$$\int \frac{x^2+3x+5}{x+1}\,\mathrm{d}x = \int x+2+\frac{3}{x+1}\,\mathrm{d}x = \frac{x^2}{2}+2x+3\ln|x+1|+c$$

Summary

Today we learned

- method of undetermined coefficients

next time, we will learn

- method of variation of parameters