# November 6 Math 2306 sec. 51 Spring 2023 Section 14: Inverse Laplace Transforms

### **Definition:**

Let f(t) be piecewise continuous on  $[0, \infty)$ . The Laplace transform of f, denoted  $\mathscr{L}{f(t)}$  or F(s) is given by.

$$\mathscr{L}{f(t)} = \int_0^\infty e^{-st} f(t) \, dt.$$

#### **Inverse Laplace Transform**

Let F(s) be a function. An **inverse Laplace transform** of F is a piecewise continuous function f(t) provided  $\mathscr{L}{f(t)} = F(s)$ . We will use the notation

$$\mathscr{L}^{-1}{F(s)} = f(t)$$
 if  $\mathscr{L}{f(t)} = F(s)$ .

### Example: Use a Table to Evaluate

(c) 
$$\mathscr{L}^{-1}\left\{\frac{s+4}{s^2-4}\right\}$$
  
Start al partial fractions.  
 $\frac{s+4}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$  Clear fractions  
 $s+4 = A(s+2) + B(s-2)$   
 $s+4 = A(s+2) + B(s-2)$   
 $s+4 = A(s+2) + B(s-2)$   
 $s+5 = 2$   $a+4 = A(s+2) + B(s-2)$   
 $6 = 4A \Rightarrow A = \frac{6}{4} = \frac{7}{2}$   
 $s=-2$   $-244 = A(s+2) + B(s-2)$   
 $a=-4B \Rightarrow B = \frac{2}{-4} = \frac{-1}{2}$ 

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 $\frac{S+4}{s^{2}-4} = \frac{\frac{3}{2}}{s-2} = \frac{\frac{1}{2}}{s+2}$ 

 $\mathcal{J}\left(\frac{s+\gamma}{s^{2}-\gamma}\right) = \mathcal{J}\left(\frac{3}{s-2} - \frac{1}{s+2}\right)$ 

 $= \frac{3}{2} \frac{1}{2} \left( \frac{1}{5 \cdot 2} \right) - \frac{1}{2} \frac{1}{2} \left( \frac{1}{5 \cdot (-2)} \right)$  $= \frac{3}{2} e^{2t} - \frac{1}{2} e^{2t}$ 

## A Look Ahead: Solving IVPs

If f(t) is defined on  $[0, \infty)$ , is differentiable, and has Laplace transform  $F(s) = \mathscr{L} \{f(t)\}$ , then\*

$$\mathscr{L}\left\{f'(t)\right\} = sF(s) - f(0)$$

Use this result to solve the initial value problem

$$y'(t) + 4y(t) = 16t$$
,  $y(0) = 1$   
Well assume that the solution  $y(t)$  has a Loploce  
trans form. Let  $Y(s) = \chi(y(t))$ .

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$$y'(t) + 4y(t) = 16t, \quad y(0) = 1$$
  
Take the brows form of both sider  

$$\mathcal{L} \{y' + 4y\} = \mathcal{L} \{16t\}$$

$$\mathcal{L} \{y'\} + 4\mathcal{L} \{y\} = 16\mathcal{L} \{t\}$$

$$Use \quad 40F \mathcal{L} \{y\} = 16\mathcal{L} \{t\}$$

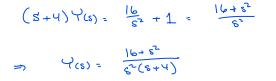
$$Use \quad 40F \mathcal{L} \{y\} = 16\mathcal{L} \{t\}$$

$$S^{2}(5) - y(6) + 4\mathcal{H}(5) = 16\left(\frac{11}{5^{2}}\right) = \frac{16}{5^{2}}$$

$$Iso [ale \quad 4(5)]$$

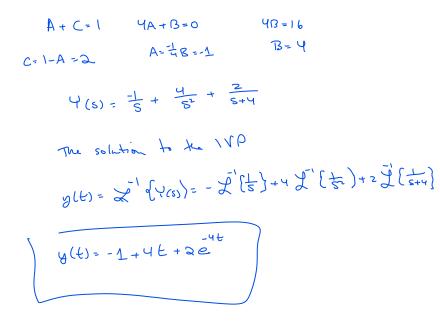
$$Stor = 1$$

$$S^{2}(5) - \mathcal{L} + 4\mathcal{H}(5) = \frac{16}{5^{2}}$$



Partial frections  

$$\begin{aligned}
\left(s^{2}(s+u)\right) & \frac{16+s^{2}}{s^{2}(s+u)} &= \frac{A}{s} + \frac{B}{s^{2}} + \frac{C}{s+u} \quad clear \quad frection \\
16+s^{2} &= A s(s+u) + B(s+u) + Cs^{2} \\
&= A(s^{2}+us) + B(s+u) + Cs^{2} \\
s^{2} + 0s + 1b &= (A+C)s^{2} + (YA+B)s + YB \\
&= Cable S^{2} + Cs^{2} = S^{2} + Cs^{2} = S^{2} + Cs^{2} = S^{2} + Cs^{2} + Cs^{2} = S^{2} + Cs^{2} + Cs^{2} = S^{2} + Cs^{2} + Cs^{2} + Cs^{2} = S^{2} + Cs^{2} + Cs^{2} + Cs^{2} = S^{2} + Cs^{2} + Cs^{2} + Cs^{2} + Cs^{2} = S^{2} + Cs^{2} + C$$



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