## October 16 Math 2306 sec. 53 Fall 2024

## **Section 10: Variation of Parameters**

$$y'' + P(x)y' + Q(x)y = g(x)$$

If  $\{y_1, y_2\}$  is a fundamental solution set for the associated homogeneous equation, then the general solution is

$$y = y_c + y_p$$
 where

$$y_c = c_1 y_1(x) + c_2 y_2(x)$$
, and  $y_p = u_1(x) y_1(x) + u_2(x) y_2(x)$ .

Letting W denote the Wronskian of  $y_1$  and  $y_2$ , the functions  $u_1$  and  $u_2$  are given by the formulas

$$u_1 = \int \frac{-y_2 g}{W} dx$$
, and  $u_2 = \int \frac{y_1 g}{W} dx$ .

## Method of Undetermined Coefficients (MUC) -vs- Variation of Parameters (VoP)

Determine which method(s) could be used to find a particular solution for each ODE.

(a) 
$$y'' + 9y = \sec^2(3x)$$

(b) 
$$y'' + 9y = x^2 \cos(3x)$$
  $\sim$  0.  $\sim$  0.  $\sim$  0.  $\sim$  0.  $\sim$  0.  $\sim$  0.

(c) 
$$y''-2y'+y=\frac{e^x}{x}$$

(d) 
$$y''-2y'+y=xe^x$$

## Example:

Find the general solution of the ODE  $y'' + y = \tan x + 2e^{2x}$ .

$$\Gamma^2 + 1 = 0 \Rightarrow \Gamma^2 = -1$$
  $\Rightarrow \Gamma = \pm \sqrt{-1}$ 

$$y'' + y = \tan x + 2e^{2x}.$$

To find yr, let yp= ye, + yez where ye, solver y"+y= dex.

y"+y= tonx and yez solver y"+y= dex.

For g.(x)= tanx, we have to use v.o.p.

y= cosx, y= sinx, gi(x)= tanx, the

We notion  $W = dx \quad \begin{cases} C_{ax} \leq inx \\ -sinx & C_{ax} \end{cases} = C_{as} \times (C_{as} \times) - (-sinx) sinx$   $= C_{as} \times + sin^{2} \times = 1$ 

yp, = U, y, + U, yz

$$u_1 = -\int \frac{g_1(x)y_2(x)}{W} dx = -\int \frac{1}{1} \frac{1}{1} dx = \int \frac{1}{1} dx$$

$$= -\int \frac{\sin^2 x}{\cos x} \, dx = -\int \frac{1 - \cos^2 x}{\cos x} \, dx$$

$$= -\int \frac{S_1 n^2 x}{Cos x} dx = -\int \frac{1 - Cos^2 x}{Cos x} dx$$

$$= \int \left(Cos x - Sec x\right) dx$$

$$u_1 = Smx - ln | Secx + tmx |$$

$$u_2 = \int \frac{gy_1}{u} dx = \int \frac{tmx Cosx}{1} dx = \int Sinx dx$$

Uz = - Cosx

$$y_{p_1} = u_1y_1 + u_2y_2$$

$$= (S.nx - ln|Secx + toux_1)Cosx + (-Cosx_1)Sinx$$

$$y_{p_1} = -Cosx_1ln|Secx_2 + toux_1$$

Find yoz that solver y"+y=2ex.

92(x)=2ex. Using M.U.C. yp=Aex.

This chores no like terms w/ yc.

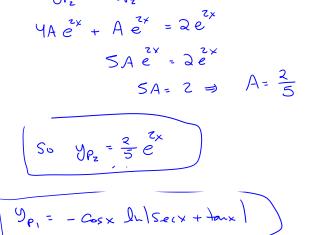
Substitute

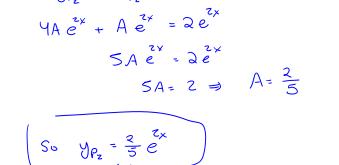
$$y_{p_{2}}^{"} + y_{p_{2}} = 2e^{2x}$$

$$y_{A}e^{2x} + Ae^{2x} = 2e^{2x}$$

$$y_{A}e^{2x} + Ae^{2x} = 2e^{2x}$$

$$y_{A}e^{2x} = 2e^{2x}$$





If we used v.o.p. to find ypz, we'd now to evaluate

$$u_1 = \int -2e^x \leq s_{nx} dx$$
 and  $v_2 = \int Ze^x cos \times dx$