## October 20 Math 2306 sec. 52 Fall 2021 Section 14: Inverse Laplace Transforms

Recall from section 13 that for *f* defined on  $[0, \infty)$ , the Laplace transform of *f* is defined by

$$\mathscr{L}{f(t)} = \int_0^\infty e^{-st} f(t) \, dt.$$

Now we wish to go *backwards*: Given F(s) can we find a function f(t) such that  $\mathscr{L}{f(t)} = F(s)$ ?

If so, we'll use the following notation

$$\mathscr{L}^{-1}{F(s)} = f(t)$$
 provided  $\mathscr{L}{f(t)} = F(s)$ .

We'll use a table to evaluate all inverse Laplace transforms.

The Laplace Transform is a Linear Transformation

Some basic results include:

$$\blacktriangleright \mathscr{L}\{\alpha f(t) + \beta g(t)\} = \alpha F(s) + \beta G(s)$$

$$\blacktriangleright \mathscr{L}{1} = \frac{1}{s}, \quad s > 0$$

• 
$$\mathscr{L}$$
{ $t^n$ } =  $\frac{n!}{s^{n+1}}$ ,  $s > 0$  for  $n = 1, 2, ...$ 

$$\blacktriangleright \mathscr{L}\{e^{at}\} = \frac{1}{s-a}, \quad s > a$$

$$\blacktriangleright \mathscr{L}\{\cos kt\} = \frac{s}{s^2 + k^2}, \quad s > 0$$

• 
$$\mathscr{L}{sin kt} = \frac{k}{s^2 + k^2}, \quad s > 0$$

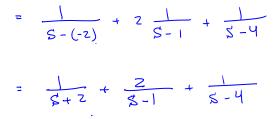
October 19, 2021 2/16

э

#### Evaluate

 $\mathscr{L}\left\{\left(e^{-t}+e^{2t}\right)^{2}\right\}$ Note that  $\left(\overset{t}{e}^{t}, \overset{zt}{e}\right)^{2} = \left(\overset{-t}{e}\right)^{2} + 2\overset{-t}{e}\overset{zt}{e} + \left(\overset{zt}{e}\right)^{2}$  $= e^{-zt} + ze^{t} + e^{-zt}$  $\mathcal{L}\left(\left(e^{t}+e^{t}\right)^{2}\right)=\mathcal{L}\left(e^{-2t}+2e^{t}+e^{4t}\right)^{2}$ =  $\mathcal{L}\left\{e^{-2t}\right\} + 2\mathcal{L}\left\{e^{t}\right\} + \mathcal{L}\left\{e^{4t}\right\}$ 

October 19, 2021 3/16

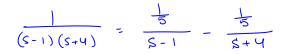


October 19, 2021 4/16

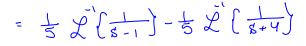
< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

## **Evaluate**

S = -4 | =  $A(0) + B(-5) \implies B = \frac{-1}{5}$ 







$$= \frac{1}{5}e^{t} - \frac{1}{5}e^{-4t}$$
$$= \frac{1}{5}e^{t} - \frac{1}{5}e^{t}$$

October 19, 2021 6/16

イロト 不得 トイヨト イヨト 二日

# Find $\mathscr{L}{f'(t)}$

Suppose f is of exponential order and has Laplace transform  $F(s) = \mathscr{L}{f(t)}$ . What is

 $\mathscr{L}{f'(t)}?$ 

meons that  $\mathcal{L}\left\{f'(t)\right\} = \int e^{st} f'(t) dt$  $= \tilde{e}^{s+} f(t) \Big|_{-}^{\infty} \int_{-s}^{\infty} \tilde{e}^{s+} f(t) dt$ 

Being of exponential orden  $\lim_{t \to \infty} -st f(t) = 0$ for s big enough Int by parts du= f'(4)2+ v = f(t) $u = e^{st}$ October 19, 2021 11/16

$$= 0 - e^{\circ}f(0) + 5 \int_{0}^{\infty} e^{5t}f(t) dt$$

$$\mathcal{L}\{f'(t)\} = -f(0) + z \mathcal{L}\{f(t)\}$$
  
 $\mathcal{L}\{f'(t)\} = z F(s) - f(0)$ 

### Solve the IVP

 $y'+4y=e^t$ , y(0)=1 transform.

- isolate (is) using algebra
- find yet as L'{ You).

 $Z(y' + 4y) = Z(e^{t})$ 

October 19, 2021 13/16

= nar

ヘロト ヘロト ヘヨト ヘヨト

$$\begin{aligned} \chi\{y\} + 4 \chi\{y\} = \chi\{e^{t}\} \\ \chi\{y\} + 4 Y(s) = \frac{1}{s-1} \\ & \chi\{y\} = sY(s) - y(o) \\ sY_{(s)} - y(o) + 4 Y_{(s)} = \frac{1}{s-1} \\ & y(o) = 1 \\ sY_{(s)} - 1 + 4 Y_{(s)} = \frac{1}{s-1} \\ & Isolate Y_{(s)} using algebra \\ & (s+4)Y_{(s)} = \frac{1}{s-1} + 1 \end{aligned}$$

$$\begin{array}{rcl} Y_{(5)} &=& \frac{1}{5-1} & + & \frac{1}{5+4} \\ Y_{(5)} &=& \frac{1}{(5-1)(5+4)} & + & \frac{1}{5+4} \\ N_{0W}, we need to take & & & & & & \\ N_{0W}, we need to take & & & & & & \\ N_{0W}, we need to take & & & & & & \\ N_{0W}, we need to take & & & & & & \\ N_{0W}, we need to take & & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & & \\ N_{0W}, we need to take & & & & \\ Y_{0W} = \frac{1}{5-1} - \frac$$

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □

$$V_{(S)} = \frac{\frac{1}{5}}{5-1} + \frac{\frac{4}{5}}{5+4}$$

The solution to the IVP  

$$y(t) = \chi' \left\{ Y(s) \right\}$$

$$= \frac{1}{5} \chi' \left\{ \frac{1}{5-1} \right\} + \frac{4}{5} \chi' \left\{ \frac{1}{5+4} \right\}$$

$$= \frac{1}{5} e^{t} + \frac{4}{5} e^{4t}$$

$$y(t) = \frac{1}{5}e^{t} + \frac{4}{5}e^{-4t}$$

October 19, 2021 16/16

୬ବ୍ଦ

◆□ → ◆□ → ◆臣 → ◆臣 → □臣