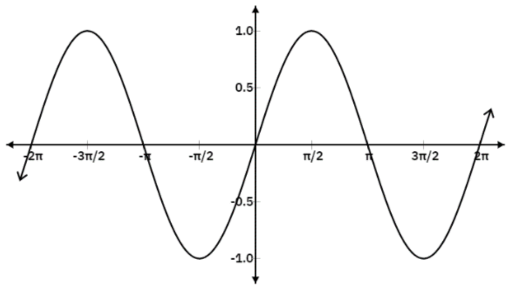
PRECALCULUS

TRIG AND INVERSE TRIG FUNCTIONS

$y = \sin x$



Domain: $(-\infty, \infty)$ or $-\infty < x < \infty$

Range: $[-1,1]$ or $-1 \le \sin x \le 1$

Period: 2π

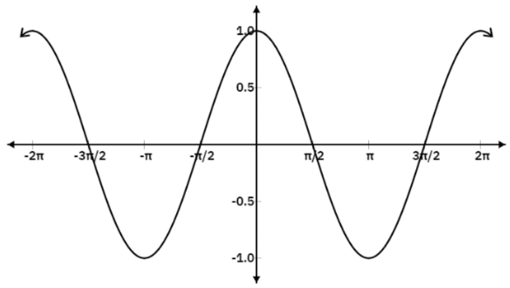
Zeros: $(k\pi, 0)$, where k is an integer

Symmetry: Odd Function

General Form: $y = a\sin(bx - c) + d$

Amplitude: $|a|$, **Period**: $\frac{2\pi}{|b|}$, **Phase Shift**: $\frac{c}{b}$

$y = \cos x$



Domain: $(-\infty, \infty)$ or $-\infty < x < \infty$

Range: $[-1,1]$ or $-1 \le \sin x \le 1$

Period: 2π

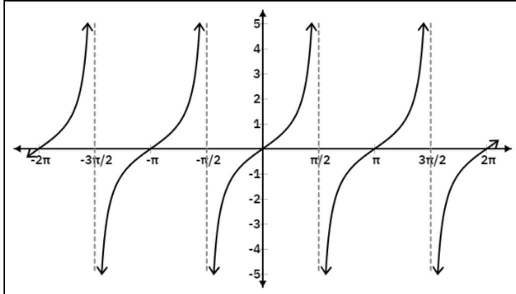
Zeros: $\left(\frac{k\pi}{2}, 0\right)$, where k is an odd integer

Symmetry: Even Function

General Form: $y = a\cos(bx - c) + d$

Amplitude: $|a|$, **Period**: $\frac{2\pi}{|b|}$, **Phase Shift**: $\frac{c}{b}$

$y = \tan x$



Domain: $\left\{x \,\middle|\, x \ne \frac{\pi}{2} + k\pi, \text{where } k \text{ is an integer}\right\}$

Range: $(-\infty, \infty)$

Vertical Asymptotes: $x = \frac{\pi}{2} + k\pi$, k an integer

Period: π

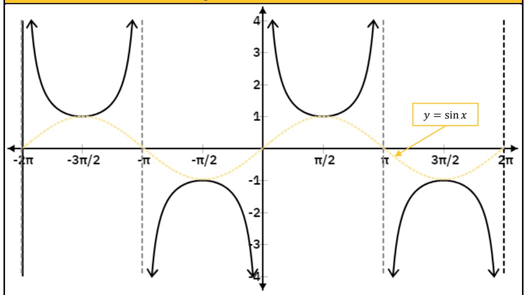
Zeros: $(k\pi, 0)$, where k is an integer

Symmetry: Odd Function

General Form: $y = a\tan(bx - c) + d$

Amplitude: $|a|$, **Period**: $\frac{\pi}{|b|}$, **Phase Shift**: $\frac{c}{b}$

$y = \csc x$



Domain: $\{x \mid x \ne k\pi, \text{where } k \text{ is an integer}\}$

Range: $(-\infty, -1] \cup [1, \infty)$

Vertical Asymptotes: $x = k\pi$, k an integer

Period: 2π

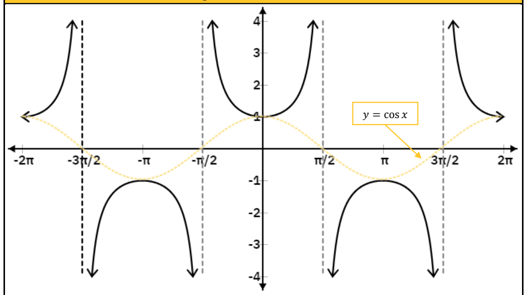
Zeros: None

Symmetry: Odd Function

General Form: $y = a\csc(bx - c) + d$

Amplitude: $|a|$, **Period**: $\frac{2\pi}{|b|}$, **Phase Shift**: $\frac{c}{b}$

$y = \sec x$



Domain: $\left\{x \,\middle|\, x \ne \frac{\pi}{2} + k\pi, \text{where } k \text{ is an integer}\right\}$

Range: $(-\infty, -1] \cup [1, \infty)$

Vertical Asymptotes: $x = \frac{\pi}{2} + k\pi$, k an integer

Period: 2π

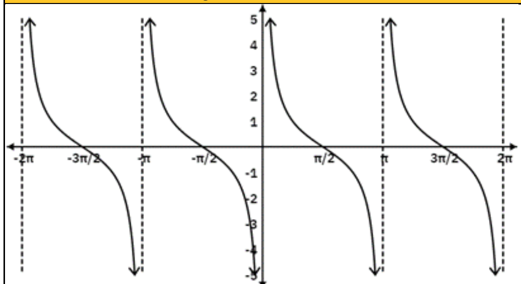
Zeros: None

Symmetry: Even Function

General Form: $y = a\sec(bx - c) + d$

Amplitude: $|a|$, **Period**: $\frac{2\pi}{|b|}$, **Phase Shift**: $\frac{c}{b}$

$y = \cot x$



Domain: $\{x \mid x \ne k\pi, \text{where } k \text{ is an integer}\}$

Range: $(-\infty, \infty)$ or $-\infty < \cot x < \infty$

Vertical Asymptotes: $x = k\pi$, k an integer

Period: π

Zeros: $\left(\frac{k\pi}{2}, 0\right)$, where k is an odd integer

Symmetry: Odd Function

General Form: $y = a\cot(bx - c) + d$

Amplitude: $|a|$, **Period**: $\frac{\pi}{|b|}$, **Phase Shift**: $\frac{c}{b}$

INVERSE TRIG DEFINITION

$y = \sin^{-1} x$ is equivalent to $\sin y = x$

$y = \cos^{-1} x$ is equivalent to $\cos y = x$

$y = \tan^{-1} x$ is equivalent to $\tan y = x$

INVERSE TRIG: CANCELLATION PROPERTIES

$\sin(\sin^{-1} x) = x$ for $-1 \le x \le 1$	$\cos(\cos^{-1} x) = x$ for $-1 \le x \le 1$	$\tan(\tan^{-1} x) = x$ for all x.
$\sin^{-1}(\sin x) = x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$\cos^{-1}(\cos x) = x$ for $0 \le x \le \pi$	$\tan^{-1}(\tan x) = x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

INVERSE TRIG FUNCTIONS: DOMAIN AND RANGE

FUNCTION	DOMAIN	RANGE
$y = \sin^{-1} x = \arcsin x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$
$y = \cos^{-1} x = \arccos x$	$-1 \le x \le 1$	$0 \le \cos^{-1} x \le \pi$
$y = \tan^{-1} x = \arctan x$	$-\infty \le x \le \infty$	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$
$y = \cot^{-1} x = \text{arccot}\, x$	$-\infty \le x \le \infty$	$0 < \cot^{-1} x < \pi$

KENNESAW STATE UNIVERSITY

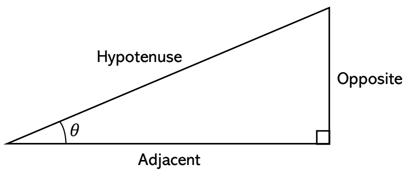
COLLEGE OF SCIENCE AND MATHEMATICS

Department of Mathematics

PRECALCULUS

TRIGONOMETRY FORMULAS

RIGHT TRIANGLE TRIG.

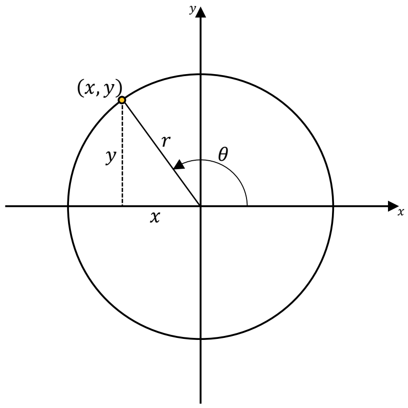


$\sin\theta = \dfrac{\text{Opp}}{\text{Hyp}}$ $\csc\theta = \dfrac{\text{Hyp}}{\text{Opp}}$

$\cos\theta = \dfrac{\text{Adj}}{\text{Hyp}}$ $\sec\theta = \dfrac{\text{Hyp}}{\text{Adj}}$

$\tan\theta = \dfrac{\text{Opp}}{\text{Adj}}$ $\cot\theta = \dfrac{\text{Adj}}{\text{Opp}}$

CIRCLE ANGLE TRIG.

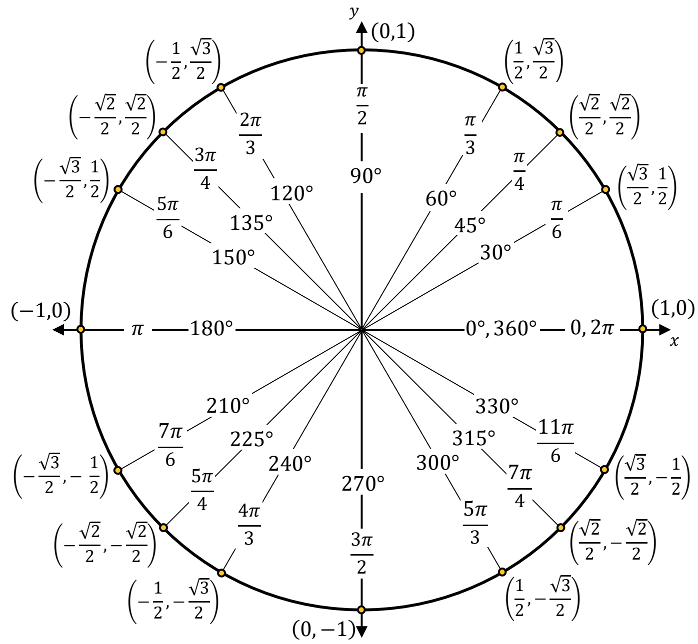


$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$

THE UNIT CIRCLE



PYTHAGOREAN THEOREM

$$x^2 + y^2 = r^2$$

OR

$$(\text{Opp})^2 + (\text{Adj})^2 = (\text{Hyp})^2$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

QUOTIENT IDENTITIES

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

RECIPROCAL IDENTITIES

$\sec\theta = \dfrac{1}{\cos\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sin\theta = \dfrac{1}{\csc\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\tan\theta = \dfrac{1}{\cos\theta}$

EVEN/ODD IDENTITIES

EVEN	ODD
$\cos(-\theta) = \cos\theta$	$\sin(-\theta) = -\sin\theta$
$\sec(-\theta) = \sec\theta$	$\csc(-\theta) = -\csc\theta$
	$\tan(-\theta) = -\tan\theta$
	$\cot(-\theta) = -\cot\theta$

DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

HALF ANGLE IDENTITIES

$$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$
$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

RADIANS ↔ DEGREES

Degrees → Radians	Radians → Degrees
$\times$ by $\dfrac{\pi}{180°}$	$\times$ by $\dfrac{180°}{\pi}$

SUM/DIFFERENCE IDENTITIES

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

PERIODIC IDENTITIES

If n is an integer

$\sin(\theta + 2\pi n) = \sin\theta$ $\csc(\theta + 2\pi n) = \csc\theta$

$\cos(\theta + 2\pi n) = \cos\theta$ $\sec(\theta + 2\pi n) = \sec\theta$

$\tan(\theta + \pi n) = \tan\theta$ $\cot(\theta + \pi n) = \cot\theta$

PRODUCT TO SUM FORMULAS

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$
$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

SUM TO PRODUCT FORMULAS

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

COFUNCTION IDENTITIES

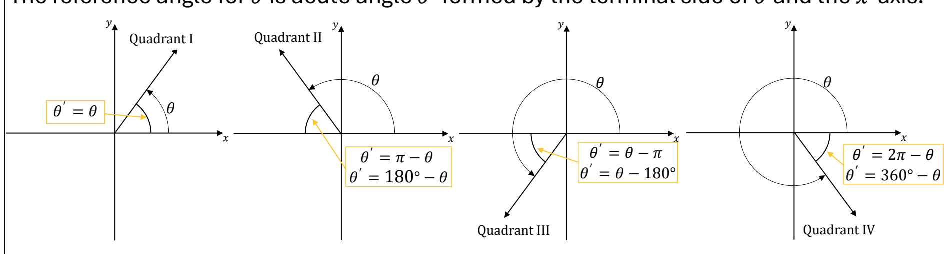
$\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$ $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$

$\sec\left(\dfrac{\pi}{2} - \theta\right) = \csc\theta$ $\csc\left(\dfrac{\pi}{2} - \theta\right) = \sec\theta$

$\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$ $\cot\left(\dfrac{\pi}{2} - \theta\right) = \tan\theta$

REFERENCE ANGLES

The reference angle for θ is acute angle θ' formed by the terminal side of θ and the x-axis.



LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$