MATH 1190 Calculus I

Section 4.3

Overview

- Curve Sketching Guidelines
- Monotonic Functions
- The Sign of the Derivative
- First Derivative Test

Announcements & Questions

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Curve Sketching Guidelines

- Identify the domain of the function f.
- Find y' and y''.
- Identify where the extrema might occur (i.e., find critical numbers).
- **Find the intervals where f is increasing and decreasing.**
- **Find any local min and max value(s).**
- Find the intervals where f is concave up and concave down.
- Find any point(s) of inflection.
- Plot some specific points (such as the local max/min, inflection points, and intercepts).
- Sketch the general shape using all of the above.

Monotonic Functions

- We say that f is **monotone** if it is increasing (or decreasing) over its *entire domain*.
  - Ex. $y = x^3$ (monotone increasing)
  - Ex. $y = -x^3$ (monotone decreasing)

- We say that f is **monotonic on** $(\boldsymbol{a}, \boldsymbol{b})$ if it is either increasing or decreasing on (a, b).

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The Sign of the Derivative

- Another corollary of the Mean Value Theorem applies to functions that are monotonic on some interval.

- **Theorem:** The Sign of the Derivative
  Let f be continuous on $[a, b]$ and differentiable on (a, b).
    - If $f'(x) > 0$ for $x \in (a, b)$, then f is increasing on (a, b).
    - If $f'(x) < 0$ for $x \in (a, b)$, then f is decreasing on (a, b).

*This notion can be extended to open intervals and infinite intervals!

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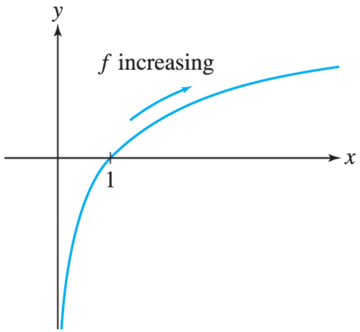
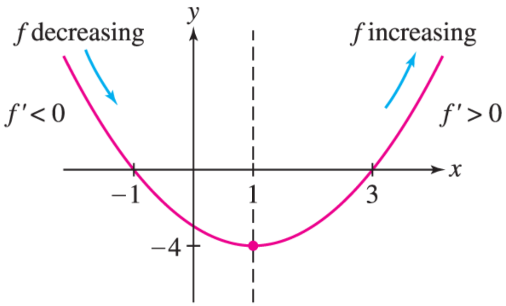
Example 0

a) To see that $f(x) = \ln x$ is increasing, observe that the derivative $f'(x) = \frac{1}{x}$ is positive on the domain $x > 0$.



b) To find the intervals on which $f(x) = x^2 - 2x - 3$ is monotonic, observe that the derivative $f'(x) = 2x - 2 = 2(x - 1)$ is positive for $x > 1$ and negative for $x < 1$.

Thus, f is increasing on $(1, \infty)$ and decreasing on $(-\infty, 1)$.



First Derivative Test

Theorem: First Derivative Test for Critical Points
Let c be a critical number of f.

- If $f'(x)$ changes sign from $+$ to $-$ at c, then $f(c)$ is a local maximum.
- If $f'(x)$ changes sign from $-$ to $+$ at c, then $f(c)$ is a local minimum.

Note 1: If f' does *not* change sign at c, then $f(c)$ is *not* an extremum.

Note 2: As long as f is continuous, then f' cannot change sign between consecutive critical numbers. That is, critical numbers subdivide the domain of f into monotonic subintervals.

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How to find local extrema of a function

1. Find the critical numbers c of f.
   - These subdivide the domain of f into monotonic subintervals.
2. Determine the Sign of the Derivative (between the critical numbers).
   - The sign of f' on each subinterval is the same for every x inside the subinterval, so pick a convenient test point x_0 and find the sign of $f'(x_0)$.
3. Determine the local extrema using the First Derivative Test.
   - The sign of f' must change sign at c for an extremum $f(c)$ to exist.

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Example 1

Find the local extrema of $f(x) = x^4 - 2x^3$.

Example 2

Find the local extrema of $g(x) = (2 - \sqrt{x})^2$.

Example 3

Find the local extrema of $f(x) = x^{1/3}(x+8)$.

Solo Practice 4.3

Find the local extrema of $g(x) = 2x^3 - 6x$.