MATH 1190 Calculus I

Section 4.4

Overview

- Curve Sketching Guidelines
- Concavity
- Test for Concavity
- Second Derivative Test

Announcements & Questions

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Curve Sketching Guidelines

- Identify the domain of the function f.
- Find y' and y''.
- Identify where the extrema might occur (i.e., find critical numbers).
- Find the intervals where f is increasing and decreasing.
- Find any local min and max value(s).
- **Find the intervals where f is concave up and concave down.**
- **Find any point(s) of inflection.**
- Plot some specific points (such as the local max/min, inflection points, and intercepts).
- Sketch the general shape using all of the above.

Concavity

Concavity refers to how the graph “bends”. More precisely, it is defined by the interaction of f with its tangents.

Let f be a differentiable function on an open interval $I = (a, b)$. Then

- f is **concave up** on I if f lies above all its tangents on I (that is, f' is increasing on I).
- f is **concave down** on I if f lies below all its tangents on I (that is, f' is decreasing on I).

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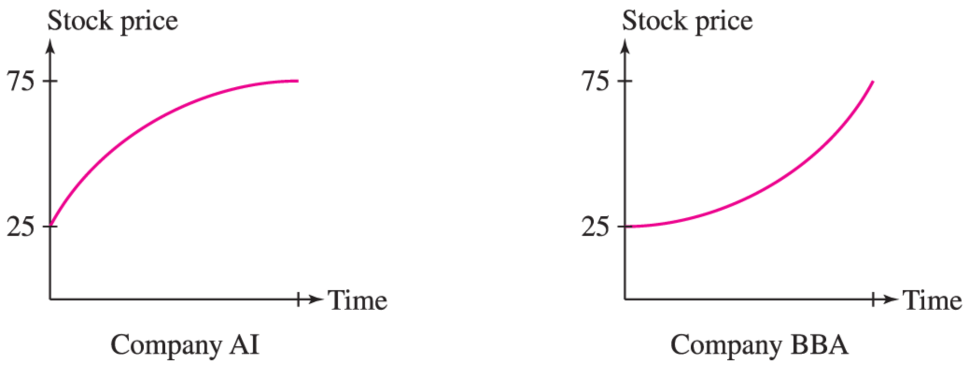
How do monotonicity and concavity interact?

	Increasing	Decreasing
Concave up		
Concave down		

Example 0

The stocks of two companies, AI and BBA, went up in value, and both currently sell for $75. However, one is clearly a better investment than the other, assuming these trends continue.

- The graph of Stock AI is concave down, so its growth rate is declining.
- The graph of Stock BBA is concave up, so its growth rate is increasing.



Test for Concavity

Theorem: Test for Concavity
Assume that $f''(x)$ exists for all $x \in (a, b)$.

- If $f''(x) > 0$ on (a, b), then f is concave up on (a, b).
- If $f''(x) < 0$ on (a, b), then f is concave down on (a, b).

A point $P\big(c, f(c)\big)$ is called a **point of inflection** (or **inflection point**) if the graph of f changes in concavity at $x = c$.

- The concavity of f is determined by the sign of $f''(x)$, so an inflection point is a point where $f''(x)$ changes sign.
- Graphically, inflection points occur where f' has a local min or max.

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How to find the inflection points of a function

Theorem: Test for Inflection Points
If $f''(c) = 0$ or $f''(c)$ does not exist (i.e., *second-order* critical numbers), and $f''(x)$ changes sign at $x = c$, then f has a point of inflection at $x = c$.

1. Find where $f''(x) = 0$ or $f''(x)$ DNE.
   - Points of inflections can *only* occur at second-order critical numbers.
2. Determine the sign of the second derivative (on each subinterval).
   - Pick a convenient test point x_0 and find the sign of $f''(x_0)$.
3. Last, determine the inflection points Test for Inflection Points.
   - The sign of f'' must change sign at c.

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Example 1

Find the inflection point(s) of $f(x) = 3x^5 - 5x^4 + 1$.

Example 2

Find the inflection point(s) of $f(x) = \cos x$ on $[0, 2\pi]$.

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Example 3

Find the inflection point(s) of $f(x) = x^{5/3}$.

How to use the curve sketching guidelines

1. Investigate the monotonicity of f:
   - Find $f'(x)$, critical numbers, and intervals of increase and decrease.
   - Determine the local extrema using the First Derivative Test.
2. Investigate the concavity of f:
   - Find $f''(x)$, second-order critical numbers, and intervals of concavity.
   - Determine the points of inflection using the Test for Inflection Points.
3. Plot local extrema and inflection points, if they exist. Use actual y-values.
4. Use the monotonicity to sketch the basic shape.
5. Use the concavity to finesse the curve.

Example 4

Sketch the curve $f(x) = 2x^3 - 6x$.

Example 5

Sketch the curve $f(x) = xe^x$.

Example 6

Sketch the curve $f(x) = \frac{4}{3}x - \tan x$ over $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Solo Practice 4.4

Sketch the curve $f(x) = \sqrt[3]{x^2 - 2x - 3}$.

Second Derivative Test (for local extrema)

There is a simple test for critical points based on concavity.

Theorem: Second Derivative Test
Let c be a critical point of f. If $f''(c)$ exists, then

- if $f''(c) > 0$, then $f(c)$ is a local minimum.
- if $f''(c) < 0$, then $f(c)$ is a local maximum.
- if $f''(c) = 0$, then f may have a local min, local max, or neither. (The test is inconclusive.)

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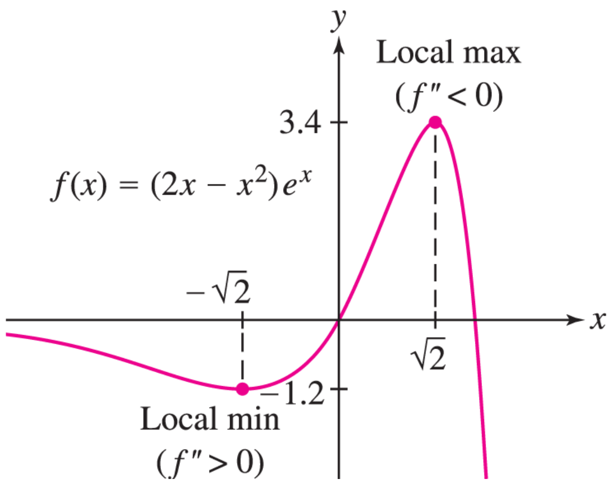
Example 7

Analyze the critical points of $f(x) = (2x - x^2)e^x$.



Example 8

Analyze the critical points of $f(x) = x^5 - 5x^4$.