

Solutions to Some Random Review of Calculus

(1) Use any appropriate derivative rules to find the derivative of the indicated function.

(a) $y = \sqrt{x^3 + 1}$ $y' = \frac{3x^2}{2\sqrt{x^3 + 1}}$

(b) $f(x) = \cot(e^{2x})$ $f'(x) = -2e^{2x} \csc^2(e^{2x})$

(c) $g(t) = t^2 \sin(t)$ $g'(t) = 2t \sin(t) + t^2 \cos(t)$

(d) $y = \sec^3(x)$ $\frac{dy}{dx} = 3 \sec^3(x) \tan(x)$

(e) $y = \ln(x^3 + 2x + 1)$ $\frac{dy}{dx} = \frac{3x^2 + 2}{x^3 + 2x + 1}$

(f) $f(x) = \frac{x}{\sin x + 1}$ $f'(x) = \frac{\sin x + 1 - x \cos x}{(\sin x + 1)^2}$

(g) $h(x) = \frac{\ln x}{x}$ $h'(x) = \frac{1 - \ln x}{x^2}$

(h) $y = \sqrt[4]{x^5}$ $y' = \frac{5}{4} \sqrt[4]{x}$

(i) $f(x) = \sin^{-1}(x+1)$ $f'(x) = \frac{1}{\sqrt{1 - (x+1)^2}}$

(j) $h(t) = (\tan^{-1} t)^2$ $h'(t) = \frac{2 \tan^{-1} t}{1+t^2}$

(k) $y = \frac{x^3 - 1}{\sqrt{x}}$ $\frac{dy}{dx} = \frac{5}{2}x^{3/2} + \frac{1}{2}x^{-3/2}$

(2) Find the first, second and third derivative of

(a) $y = xe^x$ $\frac{dy}{dx} = (x+1)e^x$, $\frac{d^2y}{dx^2} = (x+2)e^x$, $\frac{d^3y}{dx^3} = (x+3)e^x$

(b) $f(x) = \tan x$ $f'(x) = \sec^2 x$, $f''(x) = 2 \sec^2 x \tan x$, $f'''(x) = 4 \sec^2 x \tan^2 x + 2 \sec^4 x$

(3) Find the indicated derivative.

(a) $x^3y^2 = e^x + e^y$, $\frac{dy}{dx} = \frac{e^x - 3x^2y^2}{2x^3y - e^y}$

(b) $y \tan(y) = x \sin(x)$, $\frac{dy}{dx} = \frac{\sin(x) + x \cos(x)}{\tan(y) + y \sec^2(y)}$

(1) Evaluate the given integrals.

(a) given $\int_0^1 g(x) dx = 1$, and $\int_0^2 g(x) dx = 7$, evaluate $\int_1^2 g(x) dx = 6$

(b) $\int_{-1}^2 (x^2 + 3x - 1) dx = \frac{9}{2}$

(c) $\int \tan^3 \frac{x}{2} \sec^2 \frac{x}{2} dx = \frac{1}{2} \tan^4 \frac{x}{2} + C$

(d) $\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{\cos^4 2x} dx = \frac{7}{6}$

(e) $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} = \frac{1}{6}$

(f) $\int \frac{x^3}{\sqrt{x^4+1}} dx = \frac{1}{2} \sqrt{x^4+1} + C$

(g) $\int (\sec x - \csc(2x)) dx = \ln |\sec x + \tan x| + \frac{1}{2} \ln |\csc(2x) + \cot(2x)| + C$

(h) $\int_{-3}^{-1} \frac{x-4}{x^2} dx = -\ln 3 - \frac{8}{3}$

(i) $\int 3 \cot w dw = 3 \ln |\sin w| + C$

(j) $\int \left(\frac{1}{\sqrt{1-x^2}} + \frac{2}{1+4x^2} \right) dx = \sin^{-1} x + \tan^{-1}(2x) + C$

(k) $\int \frac{2x+1}{x^2+x+2} dx = \ln |x^2+x+1| + C$

(2) A particle moves along the x -axis; its acceleration $a(t)$, initial velocity $v(0)$, and initial position $s(0)$ are given by

$$a(t) = 2 \cos t \text{ ft/s}^2, \quad v(0) = 2 \text{ ft/s}, \quad \text{and} \quad s(0) = 0 \text{ ft.}$$

Find the position $s(t)$ for all $t > 0$. $s(t) = -2 \cos t + 2t + 2$

(3) Evaluate each integral using any applicable method.

(a) $\int x \sec^2 x dx = x \tan x - \ln |\sec x| + C$

(b) $\int 2xe^{x^2} dx = e^{x^2} + C$

(c) $\int 2xe^x dx = 2xe^x - 2e^x + C$

(d) $\int \sin^2 \theta d\theta = \frac{\theta}{2} - \frac{1}{4} \sin(2\theta) + C$

(e) $\int \tan^{-1} t dt = x \tan^{-1} x - \frac{1}{2} \ln |1+x^2| + C$

(f) $\int \sec^4 x \tan x dx = \frac{\sec^4 x}{4} + C = \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} + K$

(g) $\int \cos^3 t \sin^2 t dt = \frac{\sin^3 t}{3} - \frac{\sin^5 t}{5} + C$

(h) $\int \sqrt{\cot x} \csc^2 x dx = -\frac{2}{3}(\cot x)^{3/2} + C$

(4) Evaluate the integral by first using a substitution and then integration by parts.

$$\int e^{\sqrt{x}} dx = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

(5) Evaluate the given integrals using any applicable method.

(a) $\int \frac{\sqrt{x^2 - 9}}{x} dx = \sqrt{x^2 - 9} - 3 \sec^{-1} \frac{x}{3} + C$

(b) $\int \frac{dy}{\sqrt{16 - y^2}} = \sin^{-1} \frac{y}{4} + C$

(c) $\int \frac{dx}{x^2 - 1} = \frac{1}{2} \ln |x - 1| - \frac{1}{2} \ln |x + 1| + C$

(d) $\int \frac{x^3}{\sqrt{x^2 + 1}} dx = \frac{1}{3}(x^2 + 1)^{3/2} - \sqrt{x^2 + 1} + C$

(e) $\int \frac{x^2 + 7x + 2}{(x^2 + 1)(x + 3)} dx = -\ln |x + 3| + \ln |x^2 + 1| + \tan^{-1}(x) + C$

(f) $\int_0^1 \sqrt{1 - x^2} dx = \frac{\pi}{4}$

(6) Find the form of the partial fraction decomposition. (It is not necessary to find any of the coefficients A , B , etc.)

$$(a) \frac{2x}{x^2 + 7x + 12} = \frac{A}{x+3} + \frac{B}{x+4}$$

$$(b) \frac{x^2 + 2x - 1}{(x^2 - 2x + 1)(x^2 - 4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} + \frac{D}{x-2}$$

$$(c) \frac{1}{(x+2)^3(x^2-1)^2(x^2+4)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} + \frac{D}{x-1} + \frac{E}{(x-1)^2} + \frac{F}{x+1} +$$

$$+ \frac{G}{(x+1)^2} + \frac{Hx+I}{x^2+4} + \frac{Jx+K}{(x^2+4)^2} + \frac{Lx+M}{(x^2+4)^3}$$