Cubic Spline Math 2335

To build the natural cubic spline $s(x)$ to interpolate the points (x_j, y_j), $j = 1, \ldots, n$, define the constants

$$M_j = s''(x_j) \quad \text{and} \quad h_j = x_{j+1} - x_j.$$

Then the values M_j are the solutions to the system of equations:

$$M_1 = M_n = 0, \quad \text{and}$$

$$\frac{h_{j-1}}{6} M_{j-1} + \frac{h_j + h_{j-1}}{3} M_j + \frac{h_j}{6} M_{j+1} = \frac{y_{j+1} - y_j}{h_j} - \frac{y_j - y_{j-1}}{h_{j-1}}$$

$$\text{for} \quad j = 2, \ldots, n-1.$$

On each subinterval $[x_j, x_{j+1}]$

$$s(x) = \frac{M_j}{6h_j}(x_{j+1}-x)^3 + \frac{M_{j+1}}{6h_j}(x-x_j)^3 + \frac{y_j}{h_j}(x_{j+1}-x) + \frac{y_{j+1}}{h_j}(x-x_j) - \frac{h_j}{6}\left[M_j(x_{j+1}-x) + M_{j+1}(x-x_j)\right]$$

$$j = 1, \ldots, n-1.$$

Note: The conditions $M_1 = M_n = 0$ are specific to the *natural* cubic spline. If a known function f is being approximated (as opposed to raw data), additional conditions must be given to obtain equations for the M_i. See Atkinson and Han section 4.3.3, or any of the following websites for further reading:

`http://math.fullerton.edu/mathews/n2003/cubicsplinesmod.html`

`http://www.maths.lth.se/na/courses/FMN081/FMN081-06/lecture11.pdf`

Note: For equally spaced points, $h_j = h = constant$, the equations for M_j simplify to

$$M_{j-1} + 4M_j + M_{j+1} = \frac{6}{h^2}(y_{j+1} - 2y_j + y_{j-1})$$

$$\text{for} \quad j = 2, \ldots, n-1$$