Math 3260 Practice -The Fundamental Theorem of Linear Algebra

The Fundamental Theorem of Linear Algebra states that for $m \times n$ matrix A,

1. $\text{rank}(A) = \dim(\mathcal{CS}(A)) = \dim(\mathcal{RS}(A))$,
2. $\text{rank}(A) + \text{nullity}(A) = n$, and
3. $\vec{x} \cdot \vec{y} = 0$ for every vector $\vec{x} \in \mathcal{RS}(A)$ and vector $\vec{y} \in \mathcal{N}(A)$, and similarly, $\vec{u} \cdot \vec{v} = 0$ for every vector $\vec{u} \in \mathcal{CS}(A)$ and vector $\vec{v} \in \mathcal{N}(A^T)$.

Question 1. Use the Fundamental Theorem of Linear Algebra (mostly the rank-nullity theorem) to answer each question.

(a) If A is a 5×17 matrix, and $\text{rank}(A) = 3$, what is the dimension of $\mathcal{N}(A)$?

(b) If A is a 5×17 matrix, and $\text{rank}(A) = 3$, what is the dimension of $\mathcal{N}(A^T)$?

(c) If B is an 8×10 matrix, and $\dim(\mathcal{N}(B)) = 4$, how many zero rows does $\text{rref}(B)$ have?

(d) If H is a matrix with $\text{rank}(H) = 5$, and the equation $H\vec{x} = \vec{0}_8$ gives rise to four free variables, what size is H (how many rows and how many columns)?

Question 2. Let T be the set of two of vectors, $T = \{\langle 2, 1, -3\rangle, \langle 4, -2, 5\}$ in R^3, and consider the subspace $S = \text{Span}(T)$ of R^3. Find a basis for the subspace $S^\perp$, the set of all vectors in R^3 that are orthogonal to all of the vectors in S.