

# Perspective and Passion in Art, Mathematics and Pedagogy

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Why was mathematics 300 years  
behind art in the study of  
perspective and projective  
geometry?



Late 12<sup>th</sup> century Macedonia





Giotto, *Lamentation Over The Dead Christ* 1305

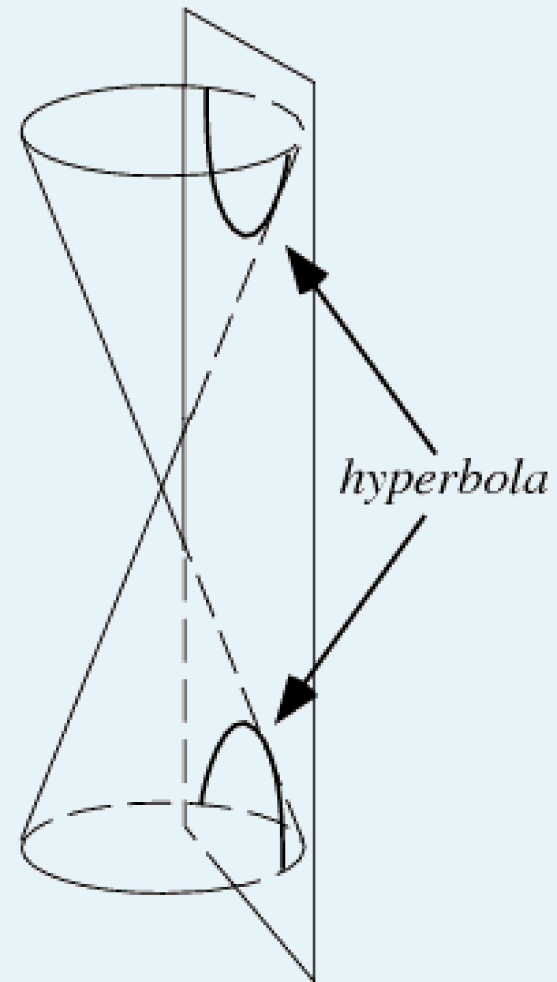
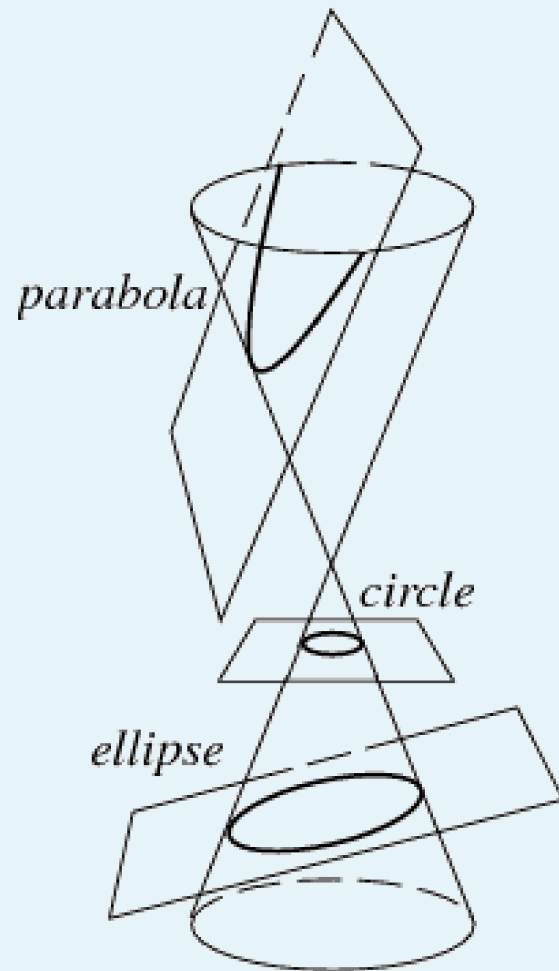




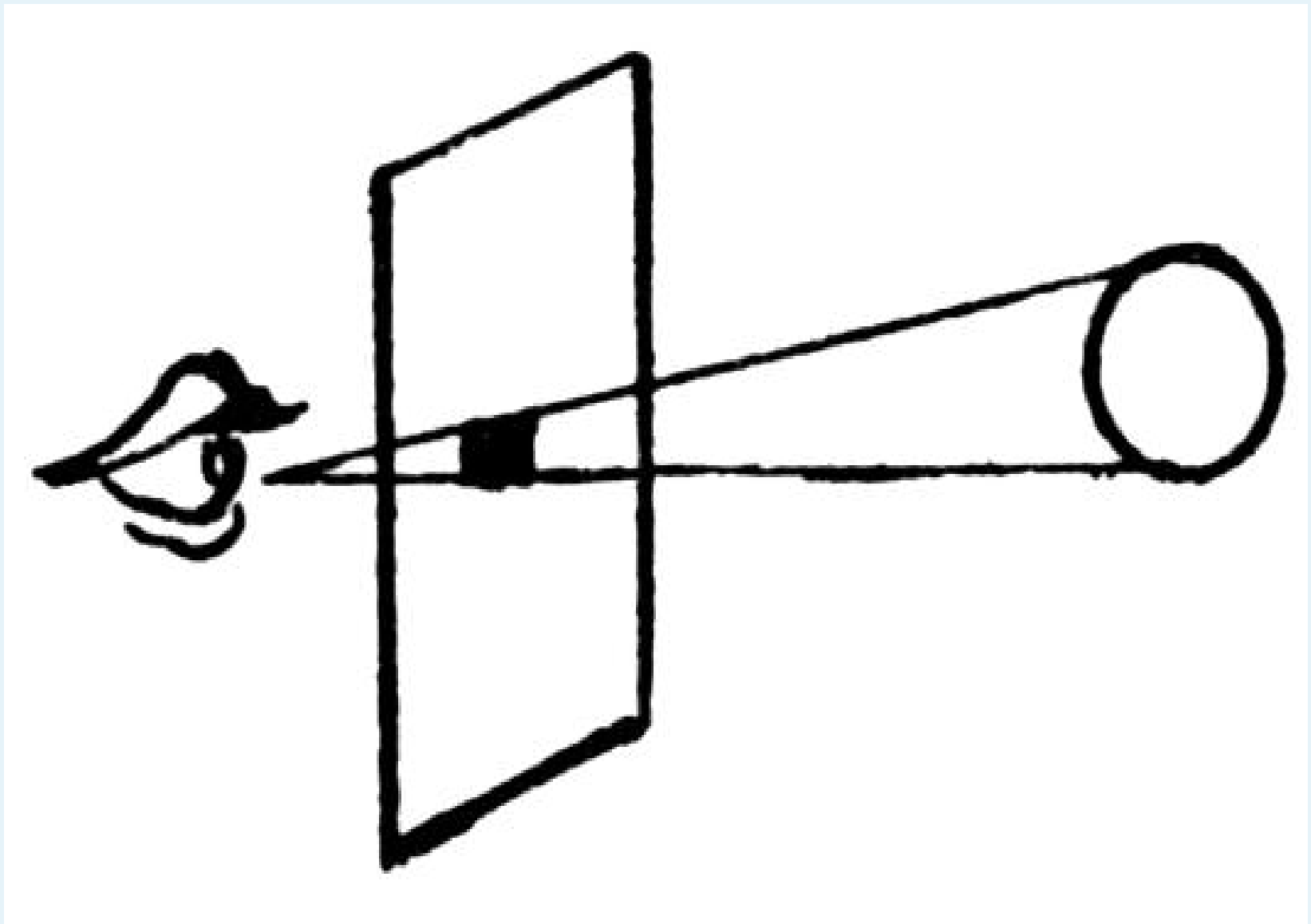
*Pentacost, The Arena Chapel*

# Rules of Perspective

- (1) The image of a straight line is a straight line
- (2) The image of a conic section is a conic section
- (3) The image of parallel lines is concurrent

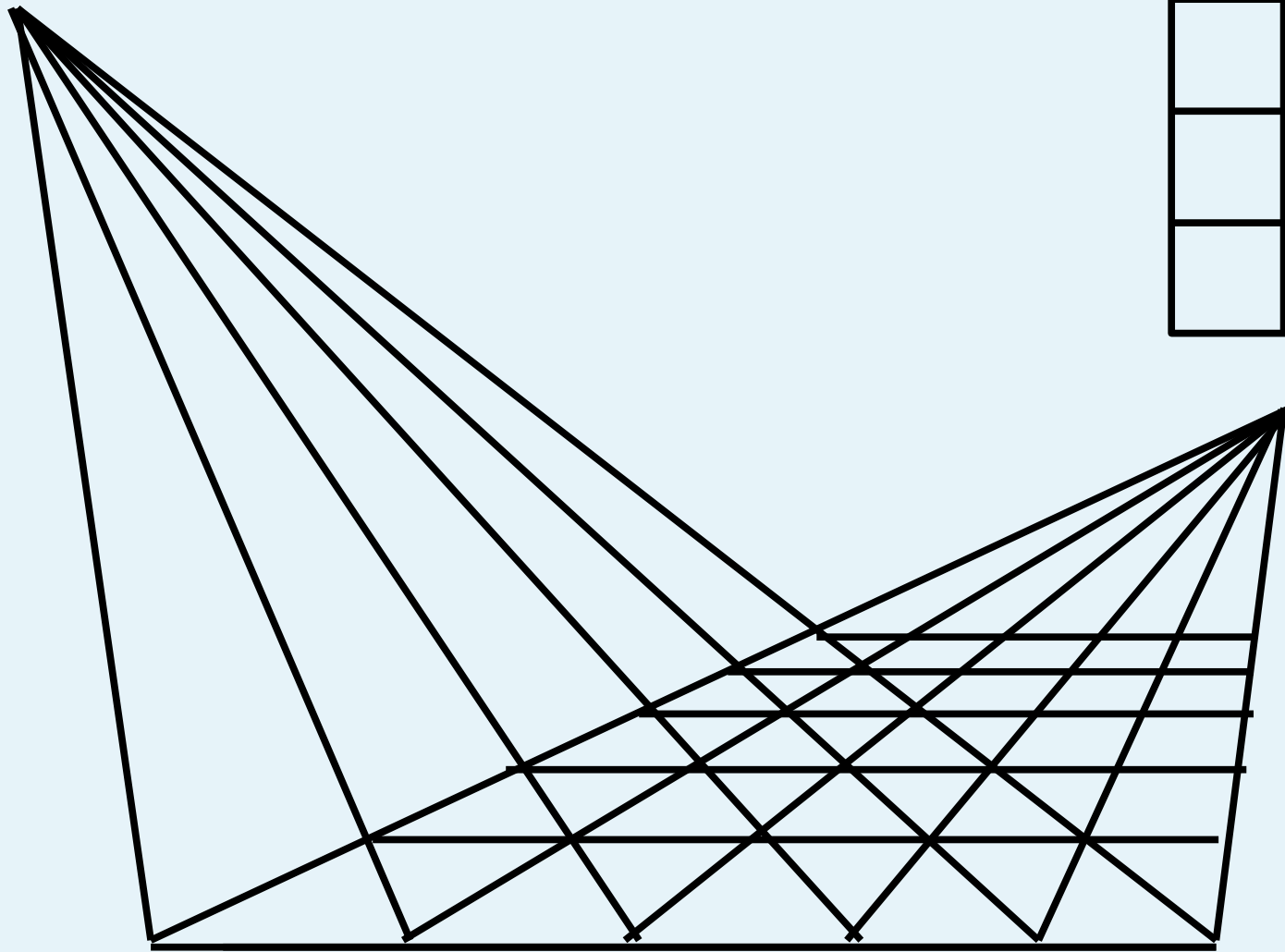


Conic Sections



The image of a conic section is a conic section.





The image of a pair of parallel lines converges.



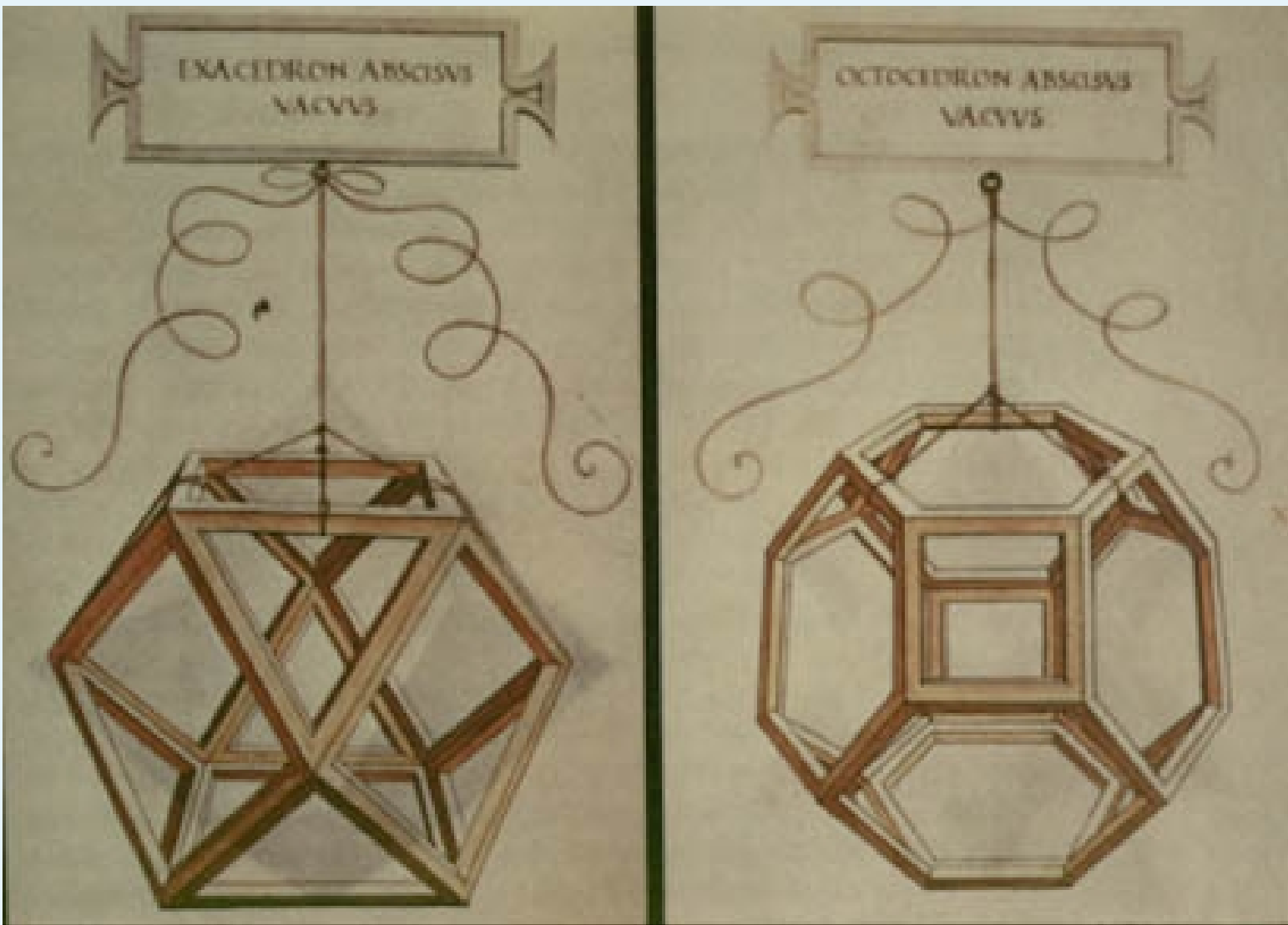
Lorenzetti, *Presentation of Christ in The Temple*, 1342



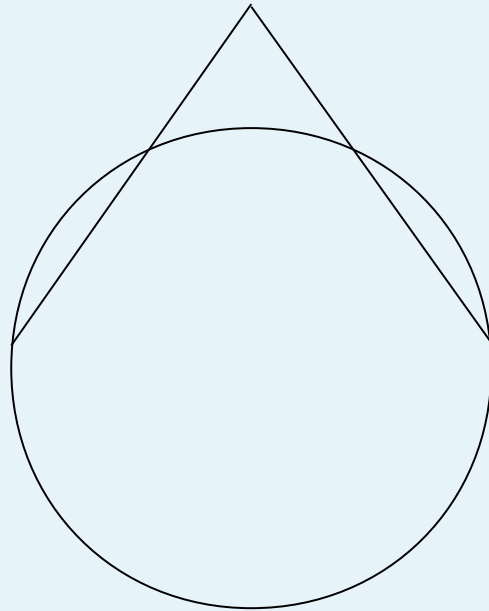


Raphael, *The School At Athens*, 1510





Leonardo's illustrations for Pacioli's *De divina proportione*, pub 1509



A cone in the drawing style used during  
most of the 15<sup>th</sup> century

# Different Agendas

**Art** What an observer sees

**Mathematics** Space as it actually is



# Artists

Abacus Schools from late 13<sup>th</sup> c.

Brunelleschi (1377-1446) Artificial Perspective

Alberti (1404-1472) *Della Pittura* (1436)

# Mathematicians

Greek Mathematics

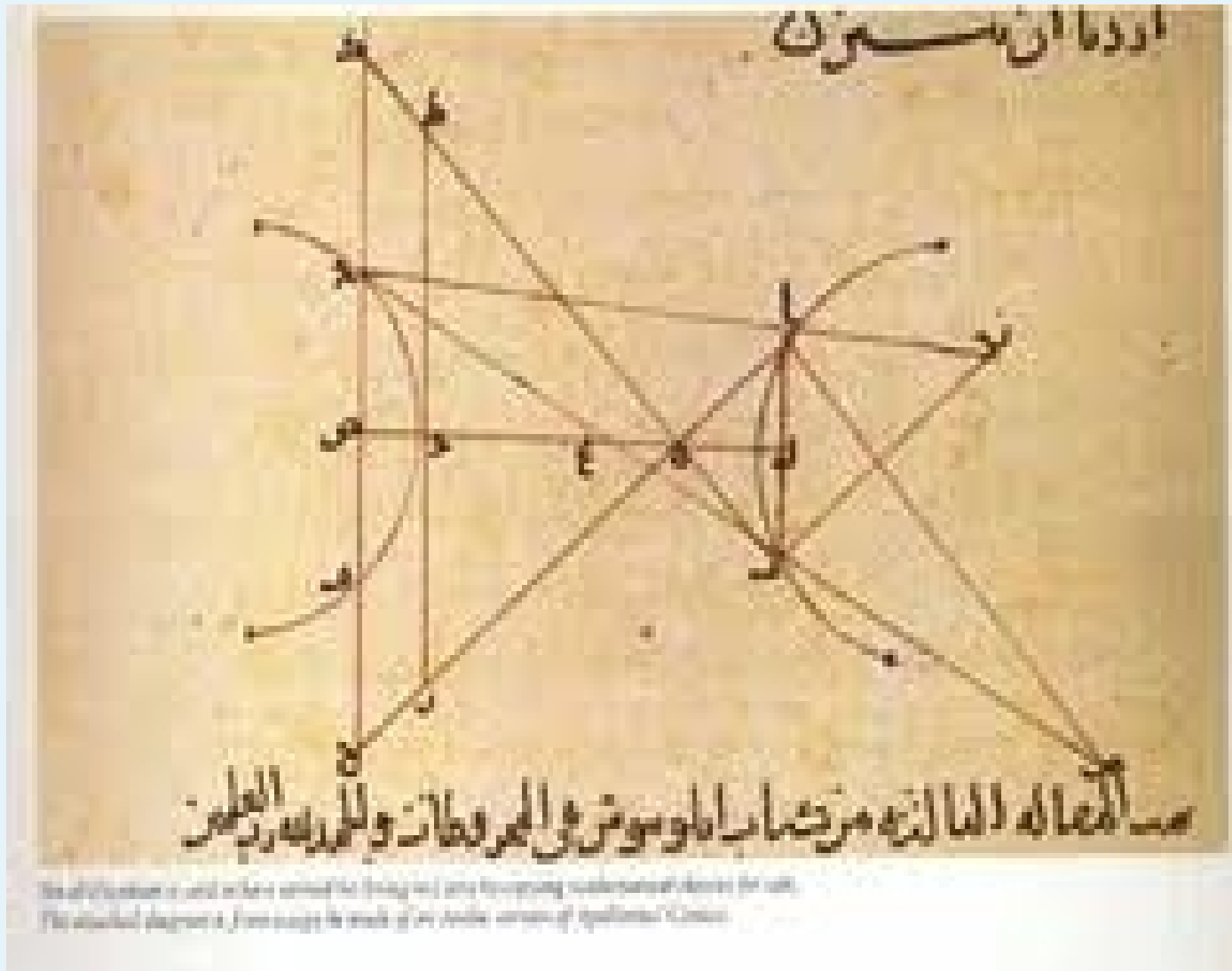
Mathematics of the Islamic Period

## From Euclid's *Optics*

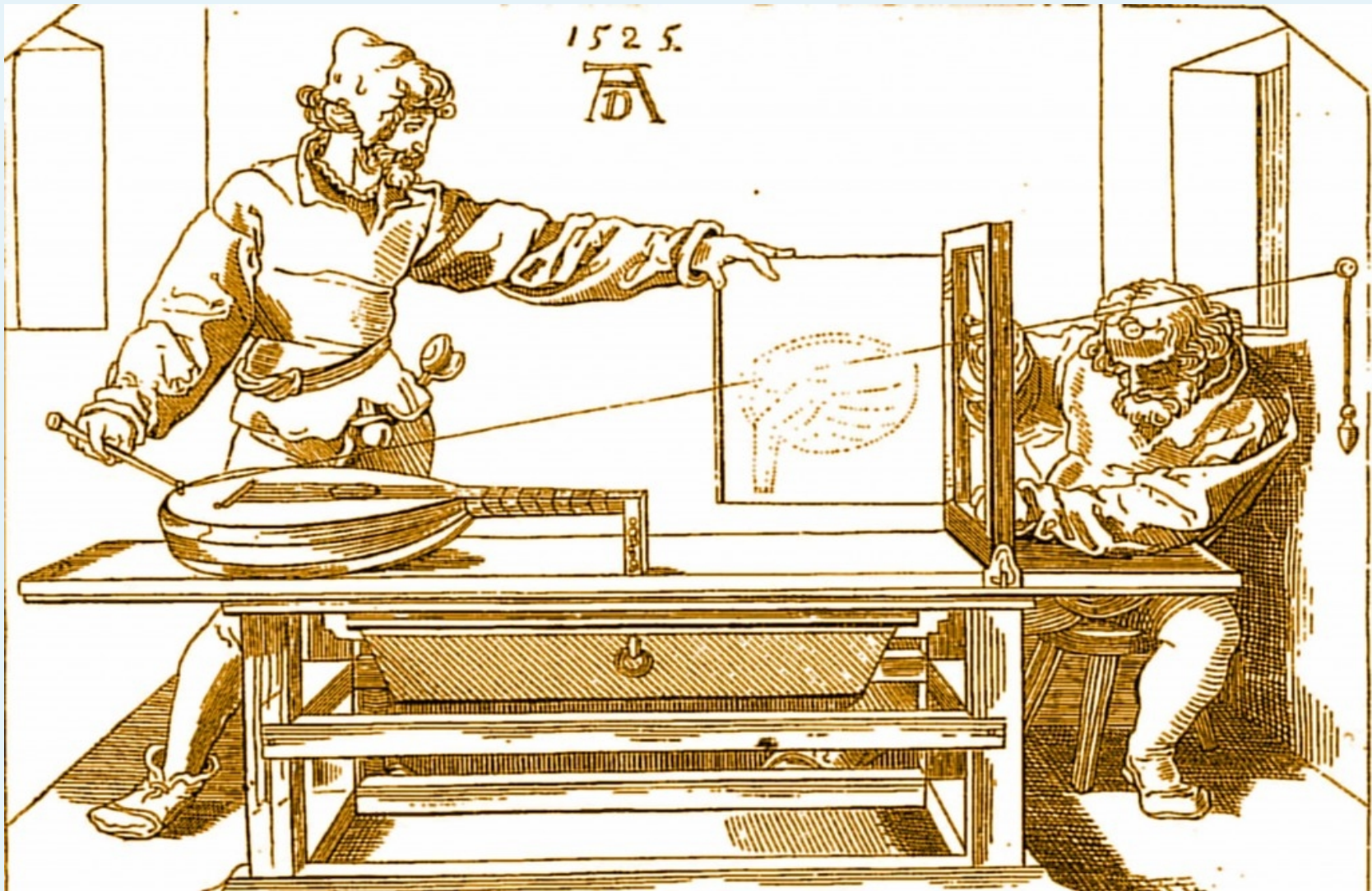
[T]he figure enclosed by the sight-lines is a cone having its vertex at the eye and its base at the limits of the things seen...

[T]hings seen by a larger angle appear larger, while things seen by a smaller angle appear smaller





Ibn al-haytham's theory of vision



Albrecht Durer's Perspective Machine



The same magnitude...viewed from near and from far does not appear equal...

***[S]cene-painting in its exploitation of this weakness of our nature falls nothing short of witchcraft***



[H]ave not measuring and numbering and weighing proved to be most gracious aids to prevent the domination in our soul of the **apparently** greater or less or more or heavier?

The part of the soul...which puts its trust in measurement and reckoning must be the best part of the soul.

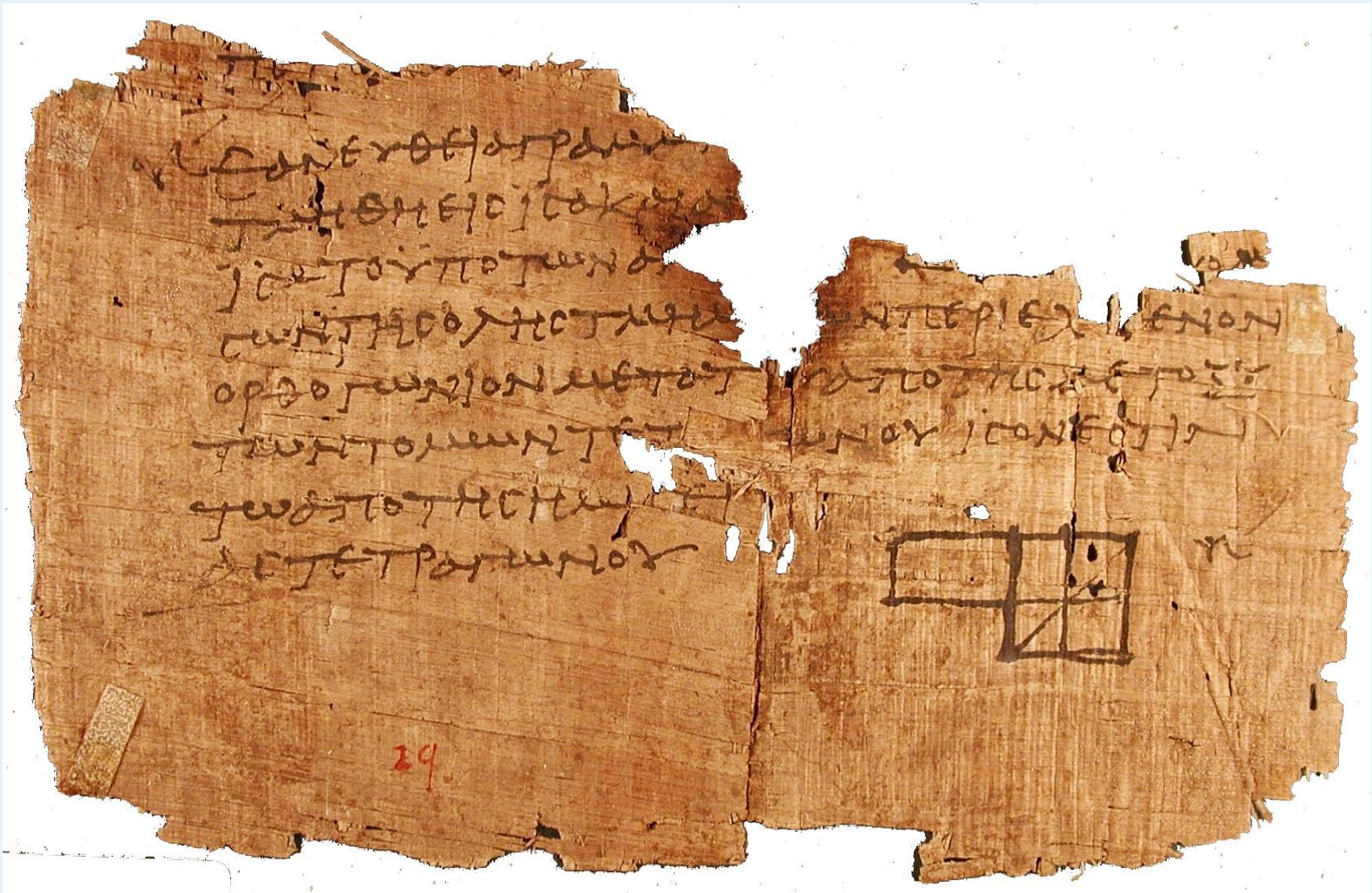
---Plato, *The Republic*

# *The Elements* of Euclid

A codification of what was known about geometry at the time, around 300 BC

The earliest known axiomatic treatment of mathematics

The standard upon which school mathematics was based since the middle of the 19<sup>th</sup> century



~100 AD Prop 5, Book II

# Book I of *The Elements*

23 Definitions

5 Postulates (axioms)

5 Common Notions

48 Propositions (theorems)

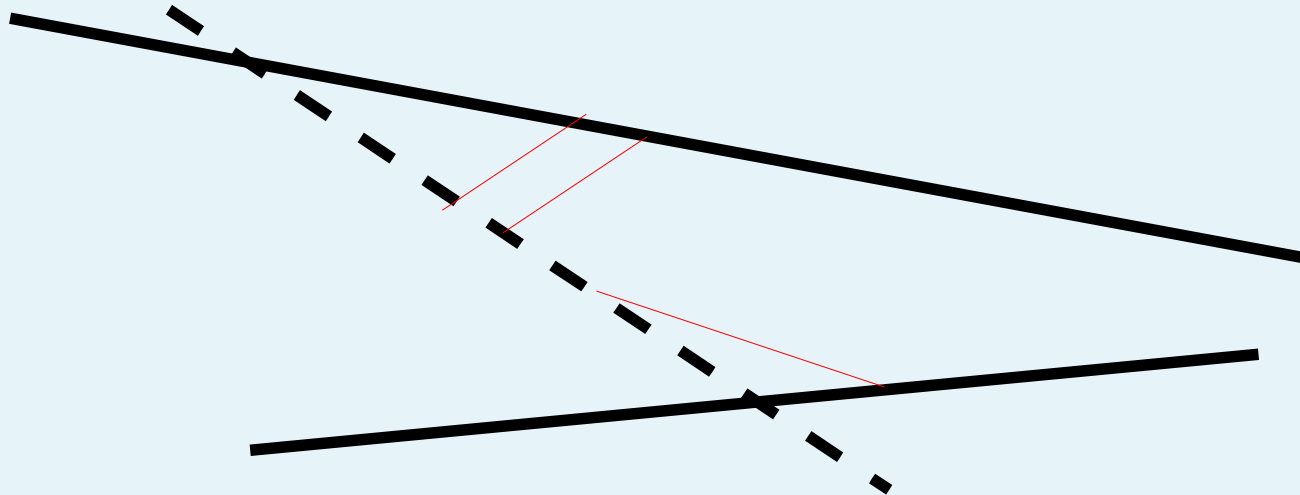


# Euclid's Postulates

1. A straight line can be drawn from any point to any point.
2. A finite straight line can be produced continuously in a straight line.
3. One may describe a circle with any center and any radius.
4. All right angles are equal to one another.

# Euclid's Fifth Postulate

That, if a straight line falling on two straight lines make an **interior angle on the same side less than two right angles**, the two straight lines, if produced indefinitely, **meet on that side** on which are the angles less than the two right angles.



# Prominent Attackers

Euclid 300 BC

Proclus 450 AD

Ibn al-Haytham 1015

Omar Khayyam 1100

John Wallis 1656

Lagrange 1776

Legendre 1800

Gauss 1817

# Equivalents to Postulate 5

1. Given a line and a point not on the line, there is exactly one parallel to the line through the point.
2. The angles of a triangle add up to two right angles.
3. There exist noncongruent similar triangles.
4. Alternate interior angles created by a transversal and two parallel lines are congruent.



**“Detest it as lewd intercourse, it can deprive you of all your leisure, your health, your rest and the whole happiness of your life.”**

Farkas Bolyai in a letter to his son János, responding to János's report that he was trying to prove the parallel postulate

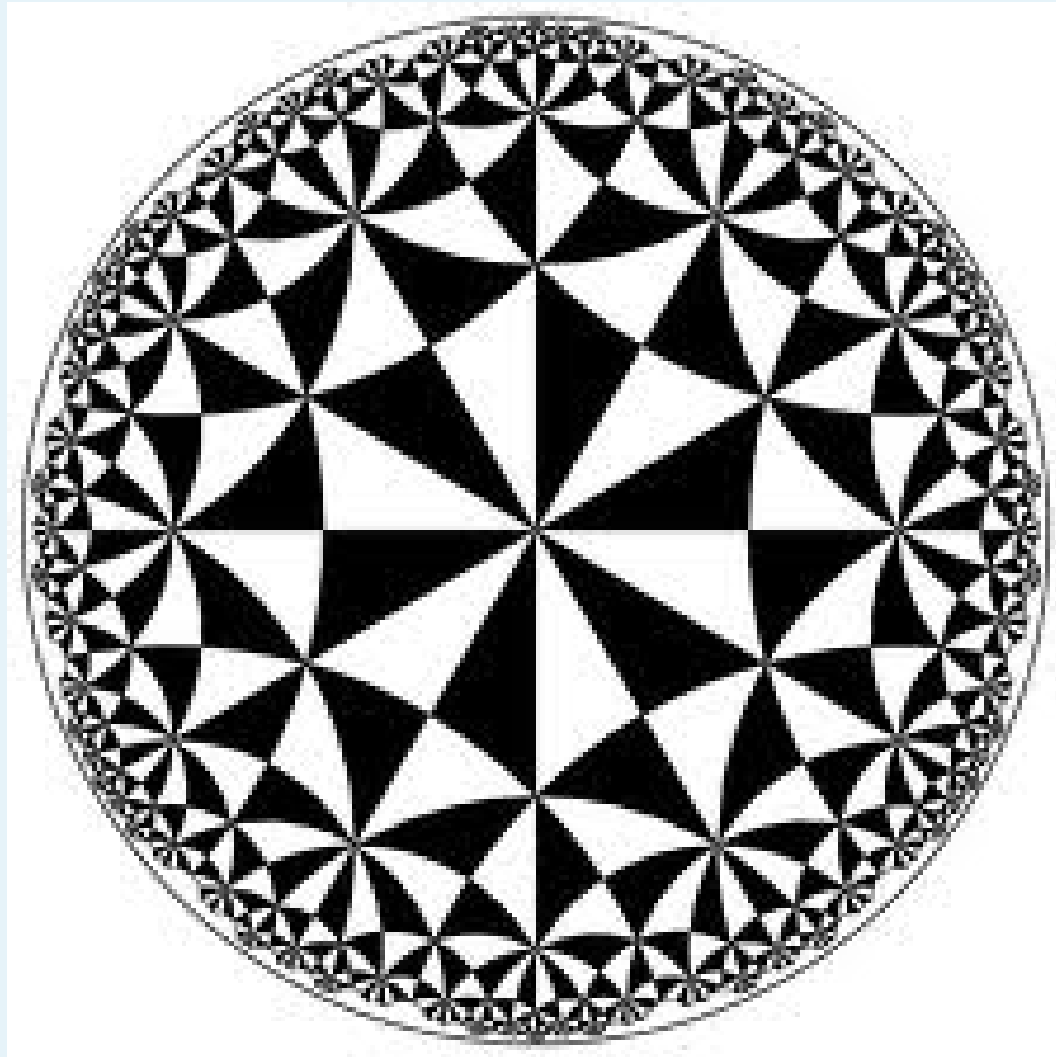
# Breakthroughs on the Parallel Postulate

Discoveries of Bolyai and Lobachevsky 1820s

Riemann's Habilitation 1854

Beltrami's interpretation 1868

# Poincaré Disk Model





# Projective Geometry as Mathematics

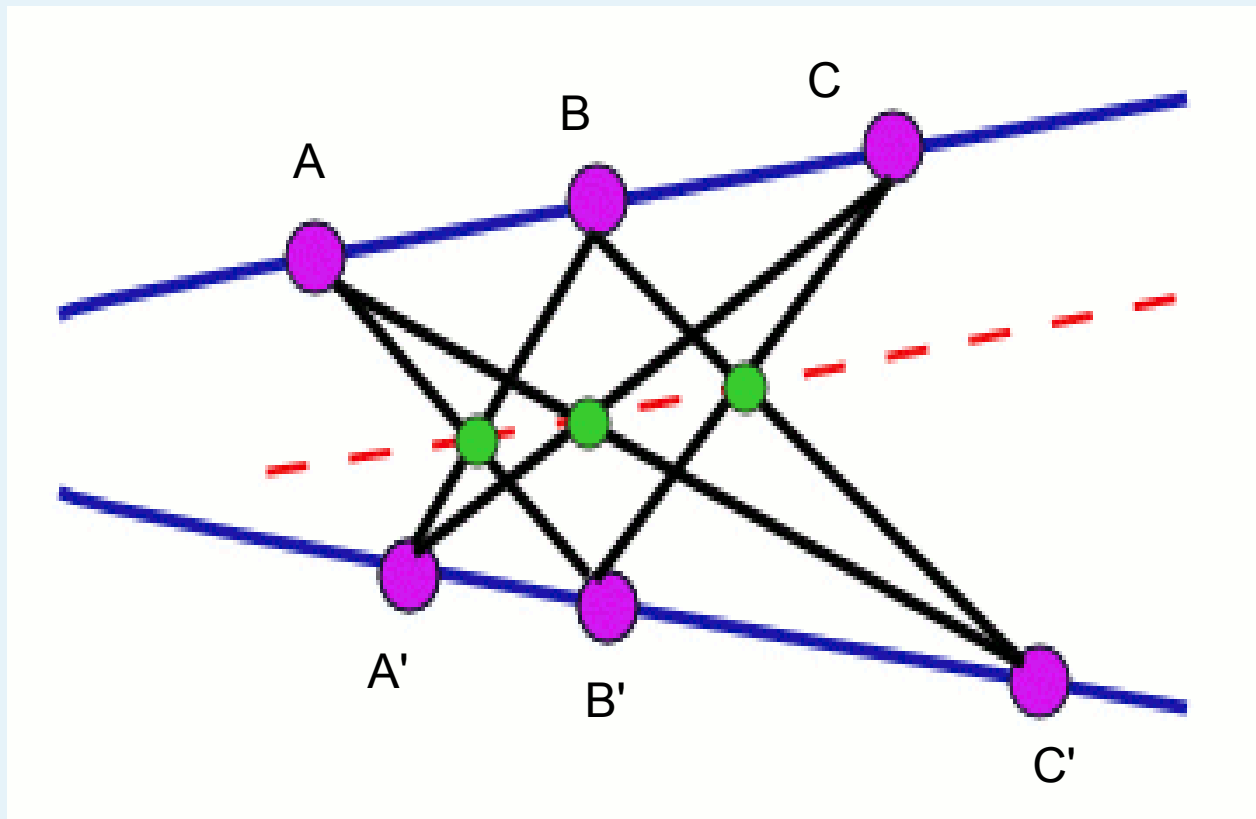
Poncelet's Treatise 1822

Plücker's homogeneous coordinates 1831

Felix Klein's algebraic foundation 1871

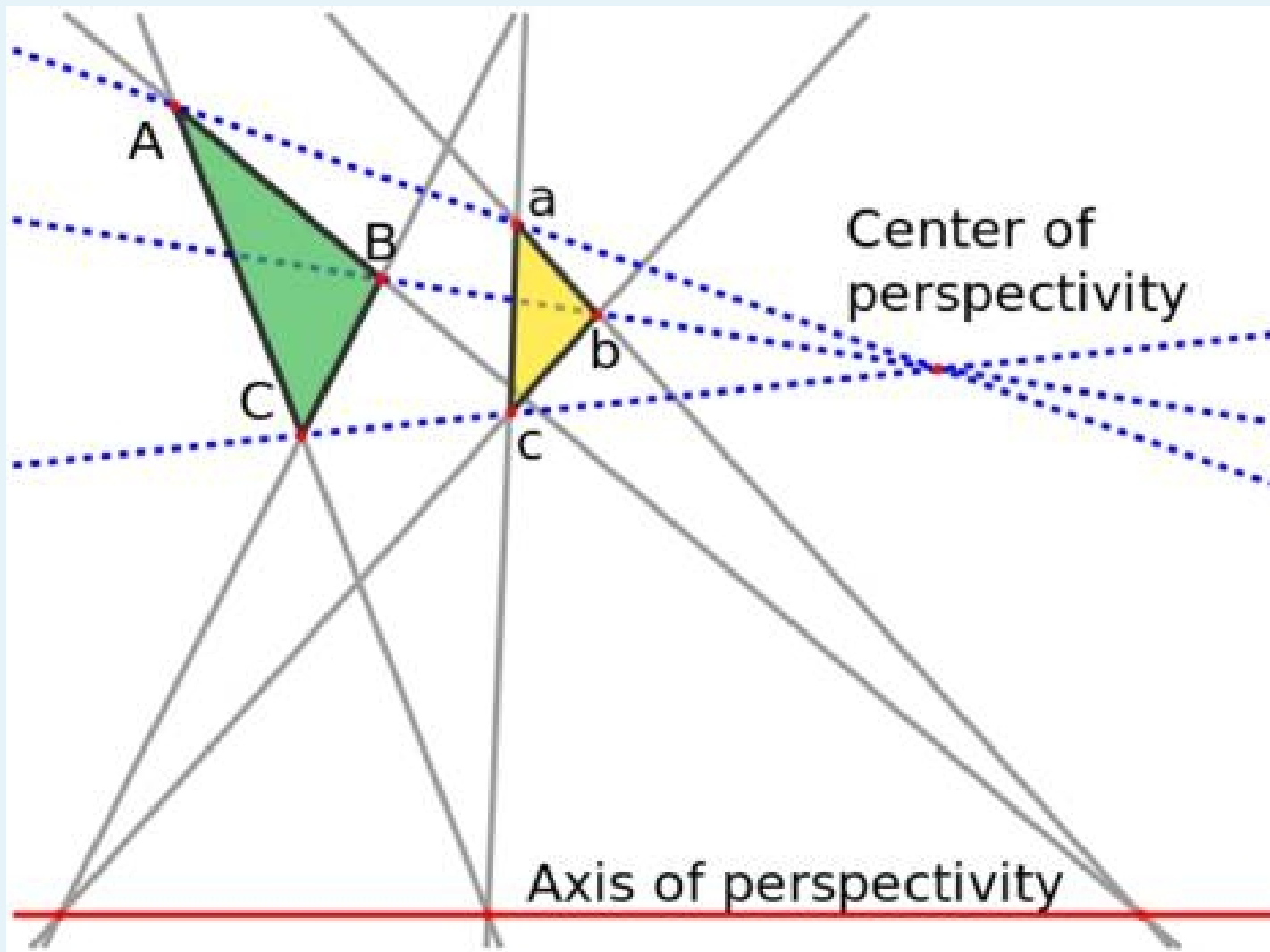
# Projective Plane

- (1) There exist four points no three of which are collinear
- (2) Two points determine a unique line
- (3) Two lines intersect in a unique point



Pappus's Theorem ~ 320 CE





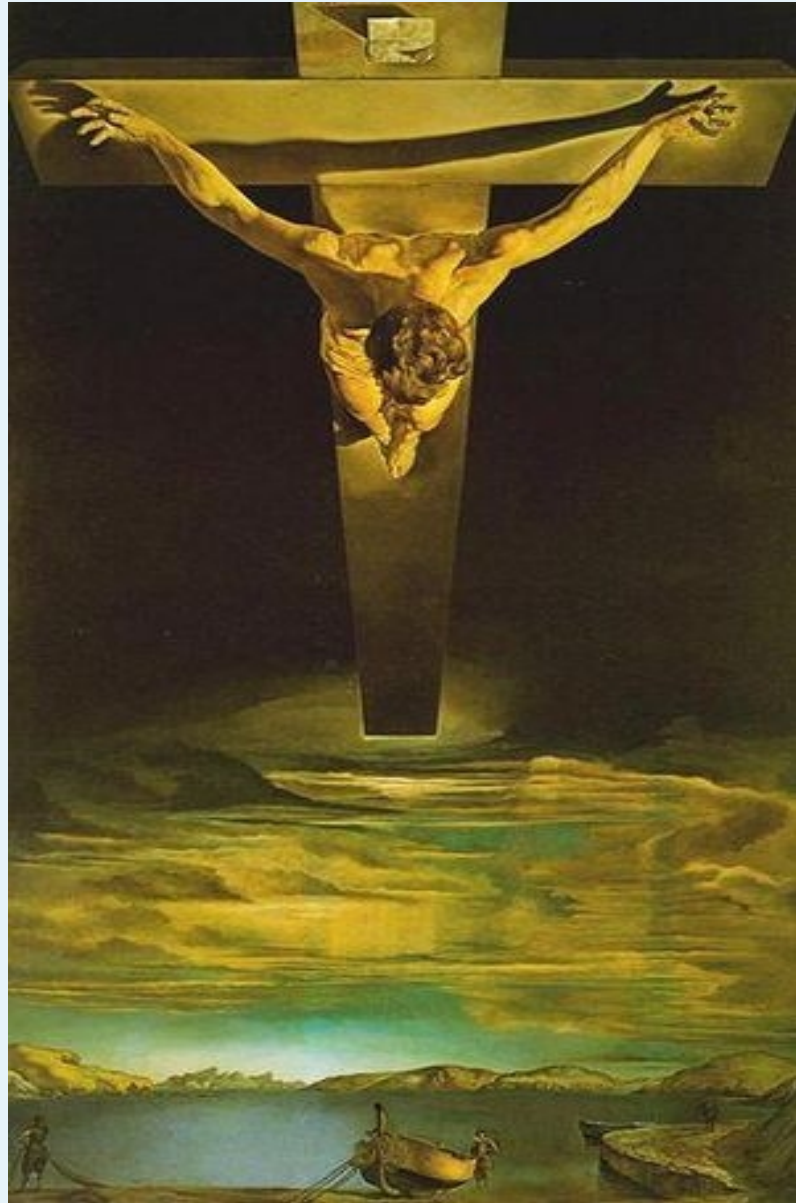
Desargues' Theorem, 1648

# What Happened Next?

Hilbert

Einstein

Bourbaki~ a return to The Elements



Dalí, *Christ of Saint John of the Cross*, 1951



**Pablo Picasso - *Nude Woman with Necklace* - 1962  
Tate, London © Succession Picasso/DACS 2007**



# Brief List of References

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Coolidge, *A History of Geometrical Methods*

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Euclid, *The Elements*, Book I, Heath edition

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MAA Monthly, Jan 2009

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Mathematicians*, American Mathematical Society, 2006.

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