Graph Theory Homework 1

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1 Short answer

In this section, all I expect you to do is find the answer, possibly accompanied by a diagram or a one-sentence explanation if I ask for it.

1. Imagine a puzzle in which four coins are lined up in a row: two pennies and two dimes. In one step, you can swap two adjacent coins.

   (a) Let G be the graph whose vertices are possible states of this puzzle, with an edge between states that are one step apart. *(Do not include "steps" that don't do anything because they swap two identical coins.)*

   Draw a diagram of G.

   (b) What is the order of G: the number of vertices?

   What is the size of G: the number of edges?

   (c) Find a cycle in G.

2. Let G be the graph whose vertices are elements of the set $\{1, 2, \ldots, 15\}$, with an edge between vertices a and b if $|a - b|$ is either 4 or 6.

   (a) Draw a diagram of G.

   (b) What are the connected components of G?

3. Of the graphs in the previous two problems, one is bipartite, and one is not.

   Demonstrate that the bipartite graph is bipartite (by finding a bipartition). Demonstrate that the other graph is not bipartite (by finding an odd cycle).

4. Find a connected 8-vertex graph which has K_4 as a subgraph, and has diameter 5. Point out the subgraph, and the two vertices at distance 5 from each other.

2 Proof

In this section, you should write a proof. Write in complete sentences and justify your logic. I am not grading the length of your proof, only its correctness, but a typical solution can be a paragraph long.

5. Prove that, for $n \geq 5$, the complement of the path graph P_n has diameter 2. Explain why your proof does not work for $n = 3$ and $n = 4$.

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 2.