

# Graph Theory Homework 2

Mikhail Lavrov

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## 1 Short answer

1. Define  $Q_d$ , as in class, to be the graph with vertex set  $\{0,1\}^d$  and an edge between any two vertices that differ in one position.

Let  $G$  be the subgraph of  $Q_d$  induced by the vertices in which the total number of 1's is either 1 or 2.

- (a) Draw a diagram of  $G$  in the case  $d = 4$ .
  - (b) Determine the degree sequence of  $G$  (in general, as a function of  $d$ ). Explain which vertices have which degrees.
2. A graph has 8 vertices and 17 edges. What are the possible values for its maximum degree? Given an example for each possible value, and explain why no other values are possible.
  3. For each of the sequences below, determine whether or not it is graphic. If it is, draw a graph with that degree sequence.
    - (a) 6, 5, 4, 4, 3, 2.
    - (b) 5, 5, 5, 3, 3, 3, 3, 3.
    - (c) 6, 6, 6, 6, 5, 3, 2, 2.

## 2 Proof

4. Prove that, for  $n \geq 5$ , the complement of the path graph  $P_n$  has diameter 2. Explain why your proof does not work for  $n = 3$  and  $n = 4$ .

*You have already written a rough draft of this solution; now, write a final draft.*

5. Prove that, for all  $n$ , there is an  $n$ -vertex graph containing a vertex of every degree between 1 and  $n - 1$ . One of these degrees will occur twice.

(It may help to try constructing this graph for a few values of  $n$ .)

*Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 3.*