Graph Theory Homework 3

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1 Short answer

1. When we're being very careful, we define the complete bipartite graph $K_{4,4}$ as the graph with vertices $\{v_1, v_2, v_3, v_4, w_1, w_2, w_3, w_4\}$ and edges $\{v_i w_j : 1 \le i, j \le 4\}$.

   Let G be the graph $K_{4,4} - \{v_1w_1, v_2w_2, v_3w_3, v_4w_4\}$: the complete bipartite graph with these four edges removed. Find an isomorphism between G and the cube graph Q_3.

2. Let T be tree whose degree sequence has the form $4, 3, 2, 1, 1, 1, \dots$ (that is, $4, 3, 2$ followed by some number of 1's).

   (a) Determine the number of 1's in the degree sequence of T.

   (b) There is more than one possibility for a tree T with this degree sequence. Give two non-isomorphic trees with this degree sequence, and explain why they are not isomorphic.

3. Count the number of spanning trees of the complete bipartite graph $K_{2,5}$.

2 Proof

4. Prove that, for all n, there is an n-vertex graph containing a vertex of every degree between 1 and $n - 1$. One of these degrees will occur twice.

   You have already written a rough draft of this solution; now, write a final draft.

5. Let G be a connected graph with n vertices and n edges. Prove that G has exactly one cycle. (That is, exactly one subgraph which is a cycle graph.)

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 4.