Graph Theory Homework 6

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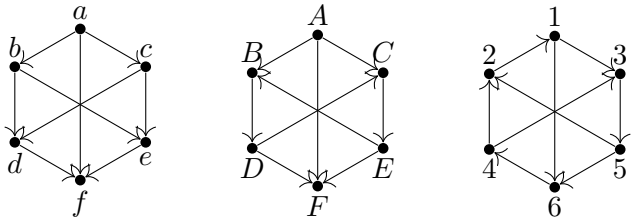
due Friday, November 5, 2021

1 Short answer

1. The three directed graphs below have the same underlying undirected graph, but the edges have different orientations.



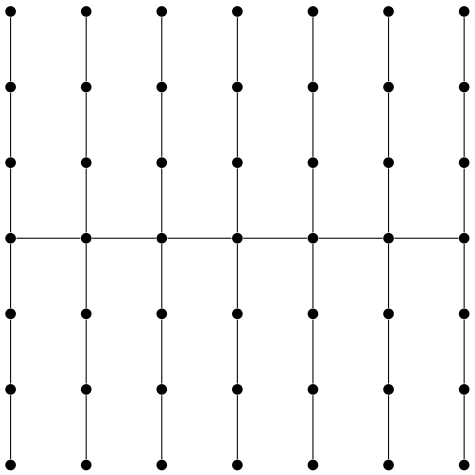
   In fact, none of the three directed graphs are isomorphic. Explain why not.

   (One way to demonstrate that two graphs are not isomorphic is to find a property that one has that the other does not.)

2. Find a tournament on 6 vertices which is not a transitive tournament (so it has cycles) but in which there is no cycle longer than 3 vertices.

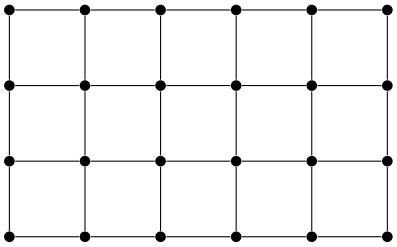
   (There are multiple solutions; you only have to find one.)

3. In the graph below, find a matching M and a vertex cover C with $|M| = |C|$.



2 Proof

4. Let G be the graph below: the "4×6 grid graph". An Eulerian tour in G would be a closed walk that uses every edge *exactly* once, but G doesn't have one of those.



Find a closed walk in G that uses every edge *at most* once, and is as long as possible (it uses as many edges as possible). Prove that your solution is optimal.

You have already written a rough draft of this solution; now, write a final draft.

5. Let G be a bipartite graph with 10 vertices on each side of the bipartition.

Prove that if G has minimum degree $\delta(G) = 3$, then G has a matching with at least 6 edges. Give an example showing that 6 might be the best possible value.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 7.