Graph Theory Homework 7

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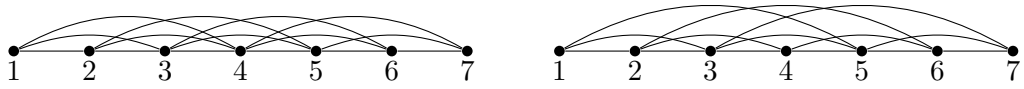
due Friday, November 19, 2021

1 Short answer

1. Exactly one of the graphs below is planar. Which one is it?



   Solve this problem either by finding a plane embedding of one of the graphs (so you know that's the planar one) or by finding an obstacle to one of the graphs being planar (so you know the other one is the planar one).

2. Suppose that a planar graph G is 4-regular and has a plane embedding with two 12-sided faces and some number of triangular faces.

   (a) Find the number of vertices in G.

   (b) Find the number of triangular faces.

3. Suppose that the complete graph K_n, with $95 < n < 105$, has a decomposition into copies of C_5. What must the value of n be?

2 Proof

4. Let G be a bipartite graph with 10 vertices on each side of the bipartition.

   Prove that if G has minimum degree $\delta(G) = 3$, then G has a matching with at least 6 edges. Give an example showing that 6 might be the best possible value.

   You have already written a rough draft of this solution; now, write a final draft.

5. Prove that an n-vertex graph G with maximum degree $\Delta(G)$ has an independent set with at least $\frac{n}{\Delta(G)+1}$ vertices.

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 8.