

Probability Theory Homework 1

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due Friday, August 27, 2021

1. The card game “Win, Lose, or Banana” is played with three cards: Win, Lose, and Banana. To play the game, you shuffle the cards and draw one to find out the outcome: you win the game, lose the game, or have a banana.
 - (a) Describe the sample space of the random experiment in which you play this game twice, recording the outcome each time.
 - (b) Express the event “you win the first game, but not the second” as a set of outcomes.
 - (c) What is the probability that you win the first game, but not the second?
2. If a number between 1000 and 9999 is chosen uniformly at random, there is a $\frac{1}{100}$ chance that it is a palindrome (like 2772, for example) and a $\frac{1}{15}$ chance that it is divisible by 15 (like $2190 = 15 \cdot 146$, for example). Only three numbers in this range are both palindromes and divisible by 15: they are 5115, 5445, and 5775.

What is the probability that a number between 1000 and 9999 chosen uniformly at random is either a palindrome or divisible by 15 (or both)?

3. You are in a game show in which you’re asked a sequence of true/false questions. If you answer two of them wrong, you’re out!! Unfortunately, you have no idea what the answers are, so you guess randomly each time, and you have a $\frac{1}{2}$ chance of guessing the correct answer.

Let A_3 be the event that your first wrong answer is to the third question you are asked.

Let B_5 be the event that your *second* wrong answer is to the fifth question you are asked.

- (a) Compute $\Pr[A_3]$.
 - (b) Compute $\Pr[B_5]$.
 - (c) Compute $\Pr[A_3 \cap B_5]$.
4. According to the schedule, a bus arrives every half hour: at 3:00pm, then at 3:30pm, then at 4:00pm, and so on. In reality, a single bus arrives at a uniformly random time between 3:00pm and 4:00pm.

Compute the probability that the bus arrives within 5 minutes (in either direction) of a scheduled arrival time.

5. Declan and Didyme each roll a fair six-sided die. Didyme’s roll is strictly larger than Declan’s roll.

Given this information, what is the probability that Didyme rolled a 6?