Probability Theory Homework 2

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1. Xavier and Yvonne compete to see who can hold their breath underwater the longest. Each of them can hold their breath for a uniformly random time between 0 and 1 minute.

   Given that Yvonne wins the contest, what is the probability that Xavier held his breadth for at least 20 seconds?

2. In **probabilistic rock-paper-scissors**, two players simultaneously choose a shape: rock, paper, or scissors. Then, the winner is chosen according to the following rules:

   - Scissors cuts paper with probability 1 (so a player playing scissors will always beat a player playing paper).

   - Rock crushes scissors with probability 0.8; with probability 0.2, scissors shatter rock (so a player playing rock will beat a player playing scissors with probability 0.8).

   - Paper covers rock with probability 0.7; with probability 0.3, rock tears paper (so a player playing paper will beat a player playing rock with probability 0.7).

   - If the two players pick the same shape, each has an equal chanch of winning.

   Peter and Sandy play this game. Peter's strategy is to pick rock and scissors with equal probability; Sandy's strategy is to pick rock and paper with equal probability. What is the probability that Peter wins?

3. A store sells light bulbs that are always last for a very long time: more than 1000 hours. However, 1 in 100 of their light bulbs are faulty: a faulty light bulb has a $\frac{1}{2}$ chance of burning out every hour.

   You buy a random light bulb from the store and test it for an hour, and it doesn't burn out. What's the probability that it's a faulty light bulb?

4. You discover that you have psychic powers! If I am thinking of a number that is 1, 2, 3, and 4, you can read my mind and correctly identify it with probability $\frac{1}{2}$. (The rest of the time, you will get one of the three wrong answers, with probability $\frac{1}{6}$ each.)

   I picked a random number x between 1 and 4. You tried to read my mind three times to figure out x. Each time, you got the result that $x = 3$. What is the probability that $x = 3$?

   (Problem 5 is on the next page.)

5. A casino offers a gambling game that's supposedly fair, but is in fact rigged to prevent winning streaks. On your first game, and on any game after you lose, you have a 50% chance of winning and a 50% chance of losing. If you win a game, you only have a 40% chance of winning, and a 60% chance of losing.

   (a) For each of $n = 1, 2, 3, 4$ compute the probability that you win your n^{th} game.

   (b) Based on your answer to (a), what is your estimate for the fraction of games you will win, if you play a long series of games?

   (Feel free to use a calculator for this one; I've set it up so that the decimals come out friendly.)