Probability Theory Homework 3

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due Friday, September 24, 2021

1. Six fair 6-sided dice are rolled.

   (a) What is the probability that *exactly* one die lands 6?

   (b) What is the probability that *at most* one die lands 6?

2. In the game Bingo, 75 balls numbered 1 through 75 are drawn at random (without replacement) out of some fancy mechanical device. Each player has a Bingo card with 24 of the 75 numbers on it.

   If you are playing Bingo, and 4 numbers have been drawn, what is the probability that exactly 2 of the numbers appear on your Bingo card?

   (a) Solve this problem using ordered sampling. That is, assume that the outcomes are ordered 4-tuples (n_1, n_2, n_3, n_4) where n_i is the ith number drawn. Find the number of outcomes where exactly 2 of the numbers appear on your Bingo card, and divide by the total number of outcomes.

   (b) Solve this problem using unordered sampling. That is, assume that the outcomes are sets $\{a, b, c, d\}$ where a, b, c, d are the four numbers drawn, with no order given. Find the number of outcomes where exactly 2 of the numbers appear on your Bingo card, and divide by the total number of outcomes.

3. Data is collected on the weather in Marietta. Each month's weather is summarized as "x sunny days, y cloudy days, z rainy days". (Each day is exactly one of sunny, cloudy, or rainy.)

   (a) Find the number of possible summaries of the weather in a 30-day month like September.

   (b) If you wanted to know the probability that there are 10 sunny days, 10 cloudy days, and 10 rainy days in September, it would not be correct to take $\frac{1}{n}$, where n is the answer to part (a). That's because we are not sampling uniformly.

   Assume that each kind of weather occurs with probability $\frac{1}{3}$, and that weather on different days is independent. Under this assumption, what is the probability that September's weather is summarized as "10 sunny days, 10 cloudy days, 10 rainy days?"

4. A fair coin is flipped 5 times. Given that it lands heads 3 times out of 5, what is the probability that the first of the five results was heads?

   (a) Without doing fancy calculations, make a reasonable guess at the answer.

(It is not $\frac{1}{2}$. If you're unsure, feel free to confirm your guess with me at any point before the homework is due.)

(b) Let A be the event "the first flip was heads" and let B be the event "3 out of the 5 flips were heads".

Use binomial probability to compute $\Pr[A \cap B]$ and $\Pr[B]$. Then, compute $\Pr[A \mid B]$ using the definition of conditional probability, hopefully confirming your answer to part (a).