# Probability Theory Homework 5

Mikhail Lavrov

due Friday, October 22, 2021

1. Let $\mathbf{X}$ be the random variable from Homework 4, Problem 2. (You can check the solutions posted on D2L to confirm your answers to that problem, or to refresh your memory if you've forgotten the distribution of $\mathbf{X}$.)

   Compute the variance $\mathrm{Var}[\mathbf{X}]$.

2. Consider a bag with marbles of three colors in it: 2 red marbles, 2 white marbles, and 2 blue marbles.

   We draw two marbles from the bag, without replacement. Let $\mathbf{R}$ be the number of red marbles we draw, and let $\mathbf{B}$ be the number of blue marbles we draw.

   (a) Find the joint probability mass function $P_{\mathbf{RB}}(k, l)$.

   You can give this as a formula in terms of $k$ and $l$, or as a table of probabilities.

   (b) Find $\Pr[\mathbf{R} + \mathbf{B} = 2]$.

3. Let $\mathbf{N} \sim \mathrm{Poisson}(4)$.

   (a) Find the conditional distribution $\mathbf{N} \mid \mathbf{N} \leq 2$. *(Note: the possible values of $\mathbf{N}$ given $\mathbf{N} \leq 2$ are 0, 1, and 2.)*

   (b) Find $\mathbb{E}[\mathbf{N} \mid \mathbf{N} \leq 2]$.

4. Suppose that random variables $\mathbf{U}$ and $\mathbf{V}$ have the joint PMF given by the following table:

| | $\mathbf{V} = 1$ | $\mathbf{V} = 2$ | $\mathbf{V} = 3$ | $\mathbf{V} = 4$ |
|---|---|---|---|---|
| $\mathbf{U} = 1$ | 1/4 | 0 | 0 | 1/4 |
| $\mathbf{U} = 2$ | 1/6 | 1/6 | 1/6 | 0 |

   Find the conditional probability $\Pr[\mathbf{U} = 1 \mid \mathbf{U} + \mathbf{V} \geq 4]$.

5. Three dice are rolled. Let $\mathbf{D}_1$ be the number of ones rolled; let $\mathbf{D}_6$ be the number of sixes rolled.

   (a) Use the formula for the variance of a binomial, find $\mathrm{Var}[\mathbf{D}_1]$, $\mathrm{Var}[\mathbf{D}_6]$, and $\mathrm{Var}[\mathbf{D}_1 + \mathbf{D}_6]$.

   (b) We could also find $\mathrm{Var}[\mathbf{D}_1 + \mathbf{D}_6]$ in terms of $\mathrm{Var}[\mathbf{D}_1]$ and $\mathrm{Var}[\mathbf{D}_6]$, if we knew the covariance. Use this rule in reverse, to find the covariance of $\mathbf{D}_1$ and $\mathbf{D}_6$.

   *(Hint: rolling a one prevents us from rolling a six with the same die, so we expect $\mathbf{D}_1$ and $\mathbf{D}_6$ to be negatively correlated.)*