Probability Theory Homework 6

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due Friday, November 5, 2021

1 Topics for exam 2

1. Each of the following random variables follows a named probability distribution we studied in class: Binomial, Geometric, Pascal, Hypergeometric, or Poisson. Name the distribution, and give its parameters.

   (a) There are billions of alien spaceships in the galaxy; on average, one alien spaceship per decade crash-lands on Earth. Let $\mathbf{A}$ be the number of alien spaceships that land on Earth in the next year.

   (b) In a class of 30 probability students, 3 are biology majors. Suppose that the instructor randomly chooses 10 different students to call on in class. Let $\mathbf{B}$ be the number of biology majors called on.

   (c) You are playing a pinball arcade machine. Each time you hit a ball with a flipper, you have a $\frac{1}{3}$ chance of losing the ball; if you don't lose the ball, it remains in play and you'll get a chance to hit it with a flipper again. You start with a supply of 3 balls, and the game ends when you run out of balls. Let $\mathbf{C}$ be the number of times you get to hit a ball with a flipper before the game ends.

2. You bring $25.00 to an arcade. You pay a quarter to play a game; each time you do, you have a $\frac{1}{10}$ chance of winning a teddy bear. You keep playing until you win the teddy bear. (Ignore the unlikely possibility of running out of money; if that happens, we'll say you'll go into debt and end with a negative amount of money.)

   Let $\mathbf{X}$ be the number of times you play the game, and let $\mathbf{Y}$ be the dollar amount you have left once you get the teddy bear.

   (a) Give the distribution of $\mathbf{X}$. Write $\mathbf{Y}$ in terms of $\mathbf{X}$.

   (b) We have formulas for the mean and variance of $\mathbf{X}$ from class. Find the mean and variance of $\mathbf{Y}$.

3. Let $\mathbf{X}$ and $\mathbf{Y}$ be independent random variables with $\mathbf{X} \sim \text{Geometric}(\frac{1}{2})$ and $\mathbf{Y} \sim \text{Geometric}(\frac{1}{3})$.

   Find the probability $\Pr[\mathbf{X} = \mathbf{Y}]$.

2 Topics not appearing on exam 2

4. Let $\mathbf{U} \sim \text{Binomial}(5, \frac{1}{2})$, $\mathbf{V} \sim \text{Binomial}(4, \frac{2}{3})$, and $\mathbf{W} = \mathbf{U} + \mathbf{V}$.

   (a) Write down $\mathbb{E}[z^{\mathbf{W}}]$ as a polynomial in factored form.

   (b) Use a computer to find the coefficient of z^3 in this polynomial. What does this coefficient tell you about $\mathbf{W}$?

   (You can use any computer software you like, but the straightforward thing is to type "coefficient of z^3 in ..." into WolframAlpha (https://www.wolframalpha.com). Please ask me if you run into technological difficulties.)

5. We will begin studying continuous random variables and their CDFs on Wednesday, November 3rd, but discrete random variables have a CDF too. The CDF (cumulative distribution function) of a random variable $\mathbf{X}$ always has the same definition: it is the function $f(t) = \Pr[\mathbf{X} \leq t]$, where t can be any real number.

   Suppose that $\mathbf{N}$ is chosen uniformly at random from the set $\{1, 2, 4, 5\}$. Graph the CDF of $\mathbf{N}$ on an interval that includes all values of t between 0 and 6.