Probability Theory Homework 7

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1. A discrete random variable $\mathbf{N}$ has the Pascal$(50, \frac{1}{2})$ distribution. For example, maybe we are flipping a fair coin, and counting the number of flips until the coin lands heads a total of 50 times.

   Use **three different techniques** that we covered in class (at any point in the semester) to put an upper bound on $\Pr[\mathbf{N} \leq 80]$.

2. A random variable $\mathbf{X}$ has a PDF of the form

   $$f_{\mathbf{X}}(t) = \begin{cases} Ct & 0 \leq t \leq 1 \\ C & 1 \leq t \leq 2 \\ C(3-t) & 2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

   (We can get such a random variable, for example, when we add an Uniform$(0,1)$ random variable to an independent Uniform$(0,2)$ random variable.)

   (a) Determine the value of C for which this is a PDF.

   (b) Find the CDF of $\mathbf{X}$.

   (c) Find $\Pr[1 \leq \mathbf{X} \leq 2]$.

3. A random variable $\mathbf{Y}$ has the PDF

   $$f_{\mathbf{Y}}(t) = \begin{cases} 3t^{-4} & t \geq 1 \\ 0 & t < 1 \end{cases}$$

   (a) Find $\mathbb{E}[\mathbf{Y}]$.

   (b) Find $\text{Var}[\mathbf{Y}]$.

4. A random variable $\mathbf{T}$ has the Exponential(10) distribution. Find $\mathbb{E}[e^{\mathbf{T}}]$.

5. If $\mathbf{U} \sim \text{Uniform}(1,3)$, find the PDF of $\frac{1}{\mathbf{U}}$.