Linear Programming Homework #1

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due Friday, August 26, 2022

1. Write a linear program for the following problem. Do not solve.

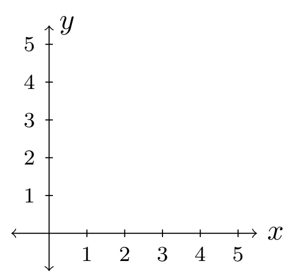
   In the tropical climate of the island country of Combinatorica, bananas can be grown spring, summer, and fall, and take only one season to grow. You want to grow x_1 cartons of bananas in the spring, x_2 cartons of bananas in the summer, and x_3 cartons of bananas in the fall, subject to the following constraints:

   - You must satisfy existing orders for 50 cartons of bananas at the end of spring, 20 cartons of bananas at the end of summer, and 80 cartons of bananas at the end of fall.
   - Unsold bananas at the end of each season can be stored indefinitely without spoiling.

   Your goal is to minimize the cost of growing bananas: \$10 per carton in the spring, \$5 per carton in the summer, and \$20 per carton in the fall.

2. Draw the feasible region for this linear program on axes like the ones shown on the right. Then solve the linear program using the naive approach.

$$\begin{array}{rrcr} \underset{x,y\in\mathbb{R}}{\text{minimize}} & x + y & & \\ \text{subject to} & 3x + 2y & \geq & 9 \\ & x - y & \geq & -2 \\ & x, y & \geq & 0 \end{array}$$



3. Convert the linear program below to equational form. Note that x and y are currently unconstrained variables (they can be positive or negative).

$$\begin{array}{rrcl} \underset{x,y\in\mathbb{R}}{\text{minimize}} & x + y & & \\ \text{subject to} & 3x - y & \geq & 2 \\ & -x + 4y & \geq & 3 \end{array}$$

4. Solve the following system of equations...

$$\begin{cases} x + y + 2z = 3 \\ 2x + y + z = 5 \end{cases}$$

(a) …for x, y in terms of z. Then, find the basic solution.

(b) …for x, z in terms of y. Then, find the solution where $y = 100$.

5. Go to https://vanderbei.princeton.edu/JAVA/pivot/simple.html. Enter the options `Constraints = 3`, `Variables = 3`, and `Seed = 7`, and click "Generate Random Problem".

   Ignore the objective function "maximize $\zeta = -2x_1 + 0x_2 - 2x_3$", as we haven't covered objective functions yet. You should see the constraints:

$$\begin{aligned} w_1 &= 5 - 5x_1 - (-1)x_2 - 5x_3 \\ w_2 &= 7 - 3x_1 - 6x_2 - (-1)x_3 \\ w_3 &= 12 - 6x_1 - (-4)x_2 - 2x_3 \end{aligned}$$

   Determine the following, starting from the dictionary above in each case:

   (a) The correct leaving variable when x_1 is the entering variable, to keep the basic solution feasible.

   (b) The correct leaving variable when x_2 is the entering variable, to keep the basic solution feasible.

   (c) The correct leaving variable when x_3 is the entering variable, to keep the basic solution feasible.

   You can use the pivot tool to check your work! Click on the x_1, x_2, or x_3 button in the row you want, and the applet will pivot to make that variable the basic variable for that row.

   In principle, I've just given you all the information you need to solve this problem without going to the applet, but this problem is also your chance to make sure the applet works for you: it's a useful tool for practice.