Linear Programming Homework #2

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1. Write a linear program for the following problem. Do not solve.

   The island country of Combinatorica wants to adopt a red, white, and blue flag. The design is unfinished: they know that they want two equal red stripes, three equal white stripes, and four equal blue stripes, but not their relative proportions. However, they want:

   - The total area of the flag to be 15 square feet.
   - Each red stripe must be at least as large as two white stripes, but no larger than three blue stripes.
   - To make sure that all colors are visible, the area of the least-represented color must be as large as possible.

   Express your linear program in terms of x_1 (the red area), x_2 (the blue area), x_3 (the white area), and x_0 (the area of the least represented color).

2. The Phase I method we discussed in Lecture 7 (Section 2.3 in Vanderbei's *Linear Programming*) can also be used to solve linear programs without an objective function: **feasibility problems**, rather than optimization problems. In such a case, the entire goal is to determine whether a system of linear inequalities has a solution, or maybe to find such a solution.

   Use this approach to find a solution to the system of inequalities

$$\begin{cases} x_1 + x_2 \le 3 \\ x_1 - 2x_2 \le -6 \\ x_1 + 2x_2 \ge 4 \end{cases}$$

   in which $x_1, x_2 \ge 0$. *(Note that the third inequality is $\ge$, not $\le$.)*

3. The linear program from `https://vanderbei.princeton.edu/JAVA/pivot/simple.html` with 3 constraints, 4 variables, and seed 108 can be solved in one pivot step. Solve it, and give the optimal solution (x_1, x_2, x_3, x_4).

4. The linear program from `https://vanderbei.princeton.edu/JAVA/pivot/simple.html` with 3 constraints, 4 variables, and seed 118 is unbounded. This can be detected after one pivot step (a second pivot step can increase the objective value without limit). Solve the linear program, and give an infinite ray of solutions along which the objective value increases without bound.

5. Consider the linear program

$$\begin{aligned} \underset{x_1,x_2,x_3\in\mathbb{R}}{\text{maximize}} \quad & x_1 + 2x_2 - x_3 \\ \text{subject to} \quad & 2x_1 - x_2 - x_3 = 3 \\ & x_1 + 2x_2 - 3x_3 = 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

(a) Set up an initial dictionary for the more complicated Phase I method discussed in Lecture 7, where we introduce artificial slack variables and minimize their sum.

(b) Solve the Phase I problem and obtain a feasible dictionary for the original linear program.