Linear Programming Homework #3

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due Friday, September 23, 2022

1. Write a linear program for the following problem. Do not solve.

   Every student at George B. Dantzig High School can join one or more of three clubs: Athletics Club, Book Club, and Chess Club. A school rule dictates that at least $\frac{1}{3}$ of the members of a club must be "dedicated members" that are not part of any other club.

   If there are 1000 students total, what is the maximum number of students that can be part of all three clubs?

2. Use the pivoting tool at https://vanderbei.princeton.edu/JAVA/pivot/simple.html to solve exercise 3.1 from the textbook with lexicographic pivoting.

   Note 1: remember that the coefficients in the dictionary are being subtracted by default if you use this pivoting tool. If you see the number -3 next to x_2 (for example), that means you are subtracting $-3x_2$, or adding $3x_2$.

   Note 2: report your optimal solution as a point (x_1, x_2, x_3, x_4) together with the objective value.

3. Consider the following linear program:

$$
\begin{array}{rl}
\underset{\mathbf{x}\in\mathbb{R}^5}{\text{minimize}} & x_1 + 4x_2 + 4x_3 + 4x_4 + x_5 \\
\text{subject to} & x_1 + 2x_2 + x_4 = 3 \\
 & x_2 + 5x_3 + 2x_5 = 1 \\
 & x_2 + 2x_3 - x_4 + x_5 = 1 \\
 & x_1, x_2, x_{3,4}, x_5 \geq 0
\end{array}
$$

   Use the formulas at the beginning of Lecture 10 to answer the questions below.

   (a) Is the basic solution with basic variables (x_1, x_3, x_5) feasible?

   (b) If the basic solution with basic variables (x_1, x_3, x_5) were feasible, would it be optimal based on the reduced costs?

   (That is, I'm asking you to ignore the judgement you made in part (a) when answering part (b). It will turn out later, however, that "Ignoring feasibility, would this basic solution be optimal based on the reduced costs?" is actually an interesting question to ask—more on this when we get to the dual simplex method!)

4. Consider the following linear program:

$$\begin{array}{ll} \underset{\mathbf{x}\in\mathbb{R}^5}{\text{maximize}} & 2x_1 + 2x_2 + x_3 + 2x_4 + 6x_5 \\ \text{subject to} & 2x_1 \quad\quad + x_3 - 3x_4 + \ x_5 = 4 \\ & \quad\quad x_2 + x_3 + 3x_4 + 2x_5 = 3 \\ & \quad\quad x_1, x_2, x_3, x_4, x_5 \ \geq 0 \end{array}$$

Compute $(A_{\mathcal{B}})^{-1}$ for $\mathcal{B} = (1, 2)$. Then, use the revised simplex method to do one pivot step. Be sure to compute the new inverse matrix in preparation for the next pivot step.

5. (a) Write the optimization problem "maximize $2x + 3y$ subject to $|x| + |y| \leq 1$, where $x, y \in \mathbb{R}$" as a linear program.

   (b) Write the optimization problem "minimize $|x| + |y|$ subject to $2x + 3y \geq 1$, where $x, y \in \mathbb{R}$" as a linear program.

   (Hint: first phrase it as minimizing z, where $z \geq |x| + |y|$.)