Linear Programming Homework #4

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due Friday, October 7, 2022

1. Write a linear program for the following problem. Do not solve. (To do the next problem, it will be helpful if you arrange your equations and inequalities to have all the variables on one side, and a constant term on the other.)

   The company "Doodads, Gizmos, and Widgets, Inc." has opened a new factory, which produces widgets, doodads, and gizmos. The factory produces exactly 100 objects per day, in any combination of widgets, gizmos, and doodads.

   Gizmos, doodads, and widgets are all produced out of raw iron, and the factory can acquire up to 2000 pounds of raw iron a day. Doodads, widgets, and gizmos all use different amounts of iron and sell for different prices:

   - It takes 10 pounds of iron to make a single doodad, which sells for a net profit of $20.
   - It takes 20 pounds of iron to make a single gizmo, which sells for a net profit of $30.
   - It takes 30 pounds of iron to make a single widget, which sells for a net profit of $40.

   What combination of gizmos, widgets, and doodads should be produced each day in order to maximize profit?

2. Write down the dual of the linear program you wrote down in problem 1. (Be sure to specify which dual variables are nonnegative, nonpositive, or unconstrained.)

3. Your friend claims that the primal linear program below has optimal solution $(x, y, z) = (5, 4, 0)$:

$$(\mathbf{P}) \begin{cases} \underset{x,y,z\in\mathbb{R}}{\text{maximize}} & x + y + z \\ \text{subject to} & 2x + y + 2z \le 14 \quad (u) \\ & x + z \le 8 \quad (v) \\ & x + 2y - z \le 18 \quad (w) \\ & x, y, z \ge 0 \end{cases} \qquad (\mathbf{D}) \begin{cases} \underset{u,v,w\in\mathbb{R}}{\text{minimize}} & 14u + 8v + 18w \\ \text{subject to} & 2u + v + 2w \ge 1 \quad (x) \\ & u + 2w \ge 1 \quad (y) \\ & 2u + v - w \ge 1 \quad (z) \\ & u, v, w \ge 0 \end{cases}$$

   The dual linear program is also given. Use complementary slackness to find the dual solution that must correspond to your friend's primal solution.

   Then, check if your friend was right that the solution $(x, y, z) = (5, 4, 0)$ is actually optimal.

4. Solve the following linear program using the **dual simplex method**:

$$\begin{array}{rrl} \underset{x,y\in\mathbb{R}}{\text{minimize}} & x + \ \ y & \\ \text{subject to} & 2x + \ \ y & \geq 3 \\ & x + 3y & \geq 4 \\ & x, y & \geq 0 \end{array}$$

Then, on a diagram of the feasible region, sketch the trajectory of the basic solutions visited by the dual simplex method on its way to the optimal solution. Here is a reference diagram of the feasible region: