Linear Programming Homework #6

Mikhail Lavrov

due Friday, November 4, 2022

1. In the game of Live-Action Rock-Paper-Scissors, both players must acquire a physical object (Rock, Paper, or Scissors) with which to play the game. The rules are still the same: rock beats scissors, scissors beats paper, and paper beats rock. However, the players must take the value of their objects into account:

   - To play Rock, it's enough to find a rock lying on the ground (for free).
   - To play Paper, a player must spend \$2 to buy a ream of paper.
   - To play Scissors, a player must spend \$3 to buy cheap plastic scissors.

   Even the most expensive purchase can be worth it, because the loser of the game must pay \$5 to the winner. (If there is a tie, neither player pays the other.)

   (a) Write down a linear program that finds the maximin strategy for this game: the mixed strategy that maximizes the expected net profit against the worst-case counter-strategy.

   (b) Is this a zero-sum game? Why or why not?

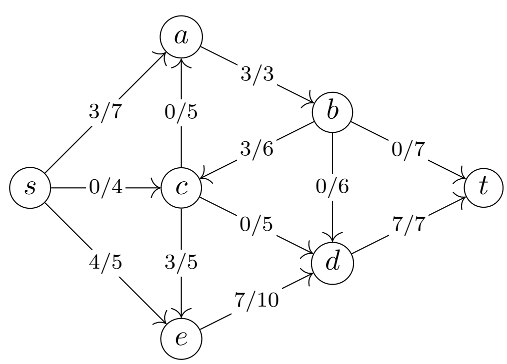
2. Find examples of networks with the following properties. For each example, describe the maximum $s-t$ flow(s) and the minimum $s-t$ cut(s).

   (a) A network with a unique maximum $s-t$ flow, but multiple minimum $s-t$ cuts.

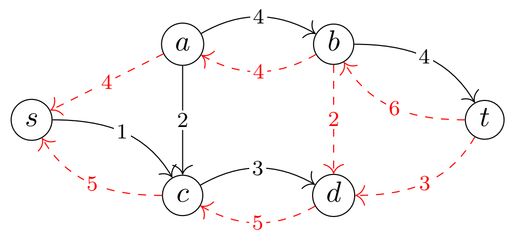
   (b) A network with multiple maximum $s-t$ flows, but a unique minimum $s-t$ cut.

   (c) A network with multiple maximum $s-t$ flows and multiple minimum $s-t$ cuts.

3. Find an augmenting path for the $s-t$ flow given in the diagram below. Then, use the path you found to augment the flow as much as possible.



4. The diagram below gives a residual graph for a network flow problem. (Solid edges are "forward" edges, dashed red edges are "backward" edges.)



   (a) Use the residual graph to reconstruct the original network, determining the edges it has and their capacities.

   (b) Find the $s - t$ flow which produces this residual graph.

   (c) Find an $s - t$ cut whose capacity is equal to the value of the flow.

5. Six universities send a few of their students to a week-long program to learn more about linear programming. There is 1 student from the first university, 2 students from the second, 3 from the third, and so on, with the sixth university sending 6 students.

   The students are divided to work on different projects. There are five projects that can accommodate up to 3 people, and one project that can accommodate any number; however, to foster inter-university collaboration, the students working on each project must all be from different universities.

   Draw a network in which a maximum $s - t$ flow can be used to determine how to assign the students to projects to satisfy these conditions. Do not solve.