Linear Programming Homework #8

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due Friday, December 2, 2022

1. In this problem, we will modify the integer program for Sudoku described in Lecture 25, which has a variable x_{ijk} for each $1 \le i, j, k \le 9$ which is equal to 1 exactly when cell (i, j) of the Sudoku grid contains digit k.

   Describe how to add constraints to the integer program to obtain **one** of the following extensions of your choice:

   (a) There is a ">" symbol between adjacent cells pointing to indicate which cell contains the greater digit. For concreteness, describe how to set up constraints so that the digit in cell $(1, 1)$ is greater than the digit in cell $(1, 2)$.

   (b) Some borders in the grid are highlighted to indicate that the two digits on either side of the border are consecutive. For concreteness, describe how to set up constraints so that cells $(1, 1)$ and $(1, 2)$ contain consecutive digits.

   (c) Some regions in the grid are highlighted and the digits within a region are required to have a specified sum, as in "Killer Sudoku". For concreteness, describe how to set up constraints so that cells $(1, 1)$ and $(1, 2)$ contain values adding up to 13.

   Again, I just want you to do **one** of these.

2. A factory produces x_1 widgets, x_2 gizmos, and x_3 doodads in a way that maximizes profit according to some constraints that we don't need to know about for this problem. Let's just assume that from these constraints, it follows that at most 1000 total items can be produced.

   Now, we want to add some additional constraints that require integer variables to formulate.

   (a) Let y_1, y_2, y_3 be binary variables: $y_1, y_2, y_3 \in \mathbb{Z}$ with $0 \le y_1, y_2, y_3 \le 1$.

   Formulate constraints that, for each i, force us to set $y_i = 1$ if $x_i > 0$.

   (b) Using the variables defined in part (a), formulate the constraint that widgets and gizmos are mutually exclusive: if the factory produces any gizmos, then it can't produce any widgets, and vice versa.

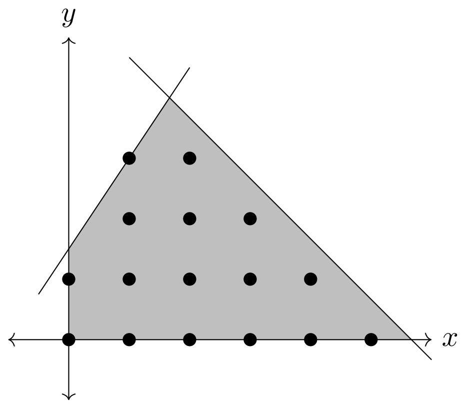
   (c) Using the variables defined in part (a), formulate the constraint that if the factory produces any doodads, then it can produce *at most* 100 widgets.

3. Solve the integer program below with the branch-and-bound method.

   For convenience, next to it is a diagram of the feasible region of the linear program and of its integer solutions, so you will not need to use the simplex method to solve subcases of the problem: you can just use the diagram to find the optimal solution. (You can still use the simplex method if you prefer.)

   However, I want you to draw me a tree diagram of the branch-and-bound method, or otherwise indicate which variables you branch on and which objective values you get at each step.

$$\begin{aligned} \underset{x,y\in\mathbb{Z}}{\text{maximize}} \quad & x + 4y \\ \text{subject to} \quad & -3x + 2y \le 3 \\ & 3x + 3y \le 17 \\ & x, y \ge 0 \end{aligned}$$



4. Solve the integer program below using the cutting plane method.

$$\begin{aligned} \underset{x,y\in\mathbb{Z}}{\text{maximize}} \quad & x + y \\ \text{subject to} \quad & x + 2y \le 7 \\ & 2x + y \le 7 \\ & x, y \ge 0 \end{aligned}$$

   (So that you don't go too far astray: you will have to add a total of two cutting planes. After each cutting plane is added, you should only have to do a single step of the dual simplex method before finding a new optimal solution. At several points in the process, you will be choosing between two equally-good options, but it shouldn't matter which you choose.)

A final note. If more than half of the students in the class submit a course evaluation before the final exam, then I will wear a turkey hat for the duration of the exam.