

Math 2390 Homework 1

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due Friday, August 26, 2022

- Write down the sets below by listing their elements:
 - $\{n^2 : n \in \mathbb{Z} \text{ and } -3 \leq n \leq 3\}$.
 - $\{n \in \mathbb{N} : 16 \text{ is divisible by } n\}$.
- Write down the sets below in set-builder notation. (There may be several reasonable ways to do this; just pick one.)
 - $\{1, 3, 5, 7, 9, 11, 13, \dots\}$.
 - $\{-36, -25, -16, -9, -4, -1\}$.

- A **proper subset** of a set S is a subset of S which is not equal to S itself. Some people write " $A \subset B$ " to mean " A is a proper subset of B ", but other people use this notation in the same way as " $A \subseteq B$ "; to be unambiguous, it is better to write " $A \subsetneq B$ " to mean " A is a proper subset of B ".

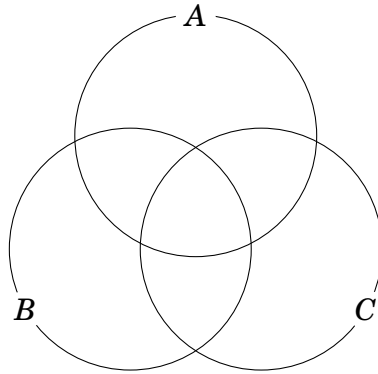
Write down all true statements of the form " $_ \subsetneq _$ " where the blanks can be filled in by \emptyset , \mathbb{Z} , $\{0, 1\}$, and $[0, 1]$.

- Find all subsets of $S = \{\emptyset, 1, \{\emptyset, 1\}\}$ which are *not* also elements of S .
(Another way of writing this problem could be "List all the elements of $\mathcal{P}(S) - S$ ".)
- Using the sets $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, and $C = \{2, 4, 6\}$ and the operations \cup (union), \cap (intersection) and $-$ (set difference), write down an expression that gives the set $\{2, 3, 4, 5\}$.
- A subset of the number line is drawn below and continues by the same pattern indefinitely to the right.



Write down an expression for this set in the form $\bigcup_{n \in \mathbb{N}} \underline{\hspace{2cm}}$.

7. Let $A = [0, 2)$, $B = (1, 3]$, and $C = (2, \infty)$. In each region of a Venn diagram such as the one drawn below, write down the set of all real numbers that belong in that region. (They are all either intervals, sets with only one element, or the empty set.)



8. Let $A = \{1, 2\}$. List the elements of $A \times \mathcal{P}(A)$.
9. Let P denote the open sentence " $x \in A$ " and let Q denote the open sentence " $x \in B$ ".
- Write " $x \in A - B$ " in terms of P and Q , using logical operations to connect them.
 - Suppose that $P \implies Q$ is true for any x . What is the relationship between sets A and B ?
10. Write the sentences below either in the form "If P , then Q " without changing the meaning.
- Only the good die young.
 - You must be this tall in order to go on this ride.
 - You can't make an omelet without breaking a few eggs.