Math 2390 Homework 3

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due Friday, September 23, 2022

1 Short answer

1. (a) Write the statement "$\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, x < y$" in words: no mathematical symbols are allowed. Is this statement true or false?

   (b) Write the statement "$\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x < y$" in words: no mathematical symbols are allowed. Is this statement true or false?

2. Come up with some reasonable mathematical notation to talk about points, lines, and lines passing through points. Then, use it to write the following two statements using quantifiers:

   (a) Through any two points, there is exactly one line.

   (b) Any two lines intersect in at most one point.

3. Consider the sentence "If $ax \equiv bx \pmod{cx}$, then $a \equiv b \pmod{c}$."

   (It's a true statement for all integers a, b, c, x where $c, x \neq 0$, but I don't want to distract you with these quantifiers in this problem.)

   (a) Rewrite the sentence without using congruence of integers, only in terms of divisibility. (Use either the notation $a \mid b$ or phrases like "a divides b" or "b is divisible by a".)

   (b) Rewrite the sentence without using congruence of integers *or* divisibility. (Expand the notation $a \mid b$ in terms of its definition.)

2 Proof

4. *For this problem, revise the rough draft you wrote for the previous assignment, based on my feedback. The result will be graded on correctness and clarity.*

Using the definition of divisibility and multiples (Definition 4.4 in the textbook), prove that every multiple of 12 is also a multiple of 3.

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

   Prove that for all integers x and y, if $x^2 + y^2 \equiv 2 \pmod 4$, then x and y must both be odd.