

Math 2390 Homework 4

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due Friday, October 7, 2022

1 Short answer

1. In this problem, assume $S, T \subseteq \mathbb{R}$. Write the following claims as logical statements beginning $\forall x \in \mathbb{R}, \dots$ and involving $x \in S$ and $x \in T$.

(e.g.) For $S \cap T \subseteq S \cup T$, you would answer: $\forall x \in \mathbb{R}, (x \in S \wedge x \in T) \implies (x \in S \vee x \in T)$.

(a) $S = (S \cap T) \cup (S - T)$.

(b) $S \cup \overline{T} \subseteq S$.

(The statement in (a) happens to be true for any S and T , and the statement in (b) is only true for some S and T , but you don't have to worry about that.)

2. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6\}$.

(a) Describe all possible functions $f: X \rightarrow Y$.

(b) Describe all possible functions $f: Y \rightarrow X$.

3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by the formula $f(x) = x^2$. Give an example of two sets $A, B \subseteq \mathbb{R}$ such that $f(A \cap B)$ is not the same thing as $f(A) \cap f(B)$.

2 Proof

4. *For this problem, revise the rough draft you wrote for the previous assignment, based on my feedback. The result will be graded on correctness and clarity.*

Prove that for all integers x and y , if $x^2 + y^2 \equiv 2 \pmod{4}$, then x and y must both be odd.

5. *For this problem, write a rough draft of a proof; any reasonable attempt will be given full credit. I will give you feedback, and you will write a final draft on the next homework assignment.*

Let A and B be arbitrary sets. Prove that if $A \times A \subseteq B \times B$, then $A \subseteq B$.