Calculus IV Homework 5

Mikhail Lavrov

due Friday, October 20, 2023

1. Let C be the curve parameterized by $\mathbf{r}(t) = (t(1-t)(1+t), t(1-t)(2-t))$ where $t \in [0,1]$. (Note that C is a closed curve: $\mathbf{r}(0) = \mathbf{r}(1) = (0,0)$.)

   Use Green's theorem to find the area of the region bounded by C.

2. Let C_1 be the curve parameterized by $\mathbf{r}(t) = (\cos^2 t, \sin t)$ where $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. Let C_2 be the line segment from $(0,1)$ to $(0,-1)$.

   (a) Points (x,y) on curve C_1 all lie on the graph of an equation $x = f(y)$ for some simple function f. What is that function?

   (b) Use Green's theorem to find the flux of $\mathbf{F} = xy^2\,\mathbf{i} + (y - e^{x^3})\,\mathbf{j}$ out of the region bounded between C_1 and C_2.

   (c) Use your answer to part (b) to find the left-to-right flux of $\mathbf{F} = xy^2\,\mathbf{i} + (y - e^{x^3})\,\mathbf{j}$ across C_1 *only* without having to take the flux integral over C_1 directly.

3. Find parameterizations of the following surfaces:

   (a) The rectangle with corners $(1,0,0)$, $(1,0,1)$, $(0,1,0)$, and $(0,1,1)$.

   (b) The portion of the sphere $x^2 + y^2 + z^2 = 1$ with $x \geq 0$.

   (c) The portion of the cone with equation $x^2 + y^2 = z^2$ bounded by $1 \leq z \leq 3$.

4. The cylinder with equation $x^2 + y^2 = 1$ bounded by $0 \leq z \leq 1$ is parameterized by

$$\mathbf{r}(u,v) = (\cos u, \sin u, v), \qquad u \in [0, 2\pi],\ v \in [0,1].$$

   Modify this parameterization in the following ways:

   (a) Rotate the cylinder to be centered around the y-axis instead: the result should be the cylinder with equation $x^2 + z^2 = 1$ bounded by $0 \leq y \leq 1$.

   (b) Shift the cylinder by 1 unit in the y-direction: the result should be the cylinder with equation $x^2 + (y-1)^2 = 1$ bounded by $0 \leq z \leq 1$.

   (c) Make the cylinder 4 times wider and 2 times longer.

5. Use an integral to find the area of the surface parameterized by

$$\mathbf{r}(u,v) = (u\cos v, u\sin v, u^2), \qquad u \in [-1,1],\ v \in [0,\pi].$$