

Calculus IV Homework 6

Mikhail Lavrov

due Friday, November 3, 2023

1. Set up, but do not evaluate, each of the following surface area integrals.

- (a) The surface area of the cylinder $x^2 + z^2 = 4$ with $-3 \leq y \leq 3$. Use a parameterization.
- (b) The surface area of the portion of the surface $z = x^2 - y^2$ satisfying $0 \leq x \leq y \leq 1$. Use the formula for a surface given by $z = h(x, y)$.
- (c) The surface area of the portion of the surface $xyz = 1$ satisfying $1 \leq x \leq 2$ and $1 \leq y \leq 2$. Use the formula for a surface implicitly given by $f(x, y, z) = 0$.

2. For each of the following shapes, describe their projections onto each of the xy -, xz -, and yz -planes, and specify which of these projections are one-to-one.

(example) The portion of the unit sphere $x^2 + y^2 + z^2 = 1$ satisfying $z \geq 0$.

The projection onto the xy -plane is a disk of radius 1, and this is a one-to-one projection. The projection onto the xz -plane is the top half of a disk of radius 1, and this projection is not one-to-one. The projection onto the yz -plane is the same as the projection onto the xz -plane.

- (a) The portion of the cone $x^2 + y^2 = z^2$ satisfying $0 \leq z \leq 1$.
- (b) The portion of the cylinder $x^2 + y^2 = 1$ satisfying $0 \leq z \leq 2$ and $y \geq 0$ (so the surface is only a half-cylinder).
- (c) The portion of the surface $z = \sin y$ satisfying $-1 \leq x \leq 1$ and $-\pi \leq y \leq \pi$.

3. Evaluate the scalar surface integral

$$\iint_S (y + z) \, d\sigma$$

where S is the surface parameterized by $\mathbf{r}(u, v) = (2uv, u^2 - v^2, u^2 + v^2)$, $(u, v) \in [0, 5] \times [0, 3]$.

- 4. Find the centroid of the portion of the sphere $x^2 + y^2 + z^2 = 1$ with $x \geq 0$, $y \geq 0$, and $z \geq 0$. (Assume uniform density.)
- 5. Let S be the surface in the shape of the parabolic bowl $z = x^2 + y^2$ with $z \leq 1$. Find the flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ across S in the *downward* direction.