Calculus IV Homework 2

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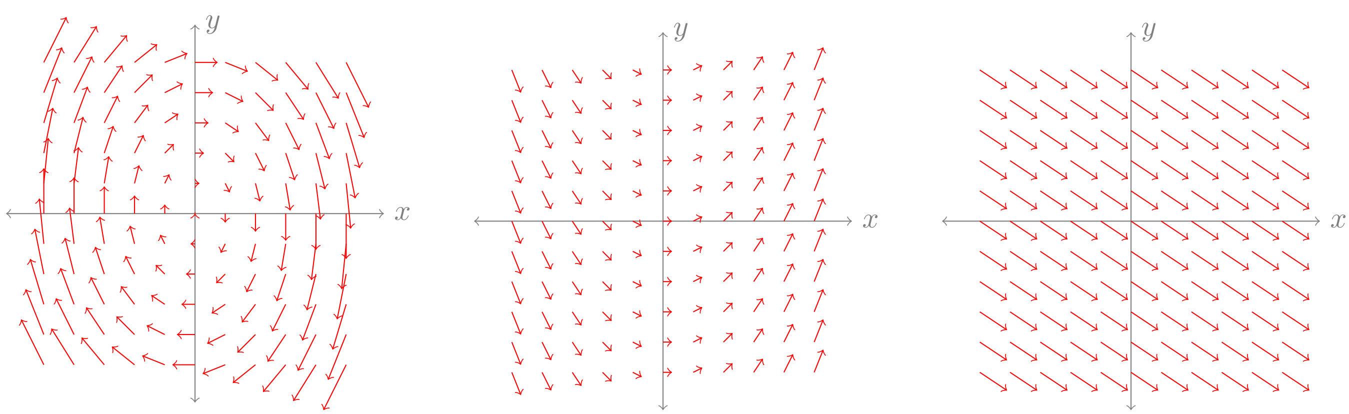
due Friday, September 6, 2024

1. Find a parameterization for each of the following curves.
   (a) The curve in $\mathbb{R}^2$ that traces the polar equation $r = \sin 2\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.
   (b) The curve in $\mathbb{R}^3$ that follows the path of a particle traveling in the plane $z = x + y + 2$ whose shadow in the xy-plane follows the unit circle.
   (c) The curve in $\mathbb{R}^3$ that goes from $(1, 1, 1)$ to $(1, -1, 1)$ along a path that lies in the intersection of the plane $x = z$ and the surface $z = y^2$.
2. For each of the following oriented curves, split it up into pieces and parameterize each piece, taking care that your parameterizations respect the orientation of the curve.
   (a) The curve in $\mathbb{R}^2$ that goes counterclockwise around the triangle with vertices $(0, 0)$, $(3, 1)$, and $(2, 3)$.
   (b) The curve in $\mathbb{R}^2$ that goes from $(-1, 1)$ to $(2, 4)$ by following the parabola $y = x^2$, then returns along a straight line.
   (c) The curve in $\mathbb{R}^3$ that goes from $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$ and back to $(1, 0, 0)$ along quarter-circles that lie on the unit sphere.
3. Integrate the scalar function $f(x, y, z) = \frac{x}{\sqrt{1+2y}}$ along the curve in $\mathbb{R}^3$ which goes from $(0, 0, 0)$ to $(1, 1, \frac{2}{3})$ parameterized by $\mathbf{r}(t) = (t, t^2, \frac{2}{3}t^3)$, where $t \in [0, 1]$.
4. Find the average distance between a point on the unit circle and the point $(1, 0)$.

   (A hint for the integral: write $1 - \cos t$ *as* $2\sin^2(t/2)$*.)*
5. Each of the diagrams on the next page is the graph of one of the vector fields

$$
\begin{aligned}
\mathbf{F}_1 &= \mathbf{i} + x\,\mathbf{j} \\
\mathbf{F}_2 &= 3\,\mathbf{i} - 2\,\mathbf{j} \\
\mathbf{F}_3 &= (x-1)^2\,\mathbf{i} + (y-1)^2\,\mathbf{j} \\
\mathbf{F}_4 &= y\,\mathbf{i} \\
\mathbf{F}_5 &= y\,\mathbf{i} - 2x\,\mathbf{j}
\end{aligned}
$$

   For each diagram, say which vector field it is the graph of (one of $\mathbf{F}_1, \ldots, \mathbf{F}_5$) and briefly explain why you think so.



(a) (b) (c)