Calculus IV Homework 3

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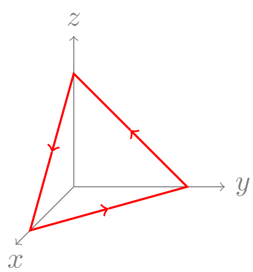
1. Let $\mathbf{F} = x^2\,\mathbf{i} - y^2\,\mathbf{k}$, and let C be the triangular path from $(1,0,0)$ to $(0,1,0)$ to $(0,0,1)$ back to $(1,0,0)$ shown in the diagram below.



   Find the circulation of $\mathbf{F}$ around C.

2. Let $\mathbf{F} = \mathbf{i} + x^2\,\mathbf{j}$, and let C be the boundary of the region $\{(x,y) : x^2 + y^2 \le 1 \text{ and } x \ge 0\}$: the right half of the unit disk. Find the outward flux of $\mathbf{F}$ across C.

3. Determine whether these vector fields are gradient fields, and if they are, find a potential function for them.

   (a) $\mathbf{F} = 4x^2y\,\mathbf{i} + \frac{4}{3}(x^3 - y^3)\,\mathbf{j}$.

   (b) $\mathbf{G} = z\cos(y+z)\,\mathbf{i} - xz\sin(y+z)\,\mathbf{j} - xz\sin(y+z)\,\mathbf{k}$.

   (c) $\mathbf{H} = (x+y)\,\mathbf{i} + (x-z)\,\mathbf{j} - (y+z)\,\mathbf{k}$.

4. (a) Let $f(x,y,z) = xe^{y-z}$. Compute the gradient field $\mathbf{F} = \nabla f$.

   (b) Let C be the coiled spring parameterized by $\mathbf{r}(t) = (\cos t, \sin t, t/\pi)$, where $t \in [0, 4\pi]$. Compute the vector line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}$ is the gradient field from part (a).

5. Let $\mathbf{F}$ be an unknown, but conservative, vector field. Define paths

   - C_1, the line segment parametrized by $\mathbf{r}_1(t) = (2t-1, 0)$, where $t \in [0,1]$.
   - C_2, the quarter-circle parametrized by $\mathbf{r}_2(t) = (\cos t, \sin t)$, where $t \in [0, \pi/2]$.
   - C_3, the parabolic curve parametrized by $\mathbf{r}_3(t) = (t^2 - 1, -t)$, where $t \in [0,1]$.
   - C_4, the cubic curve parametrized by $\mathbf{r}_4(t) = (t^3 - t, t)$, where $t \in [-1,1]$.

   If $\displaystyle\int_{C_1} \mathbf{F} \cdot dr_1 = 2$, $\displaystyle\int_{C_2} \mathbf{F} \cdot dr_2 = 3$, and $\displaystyle\int_{C_3} \mathbf{F} \cdot dr_3 = 4$, find $\displaystyle\int_{C_4} \mathbf{F} \cdot dr_4$.