Calculus IV Homework 4

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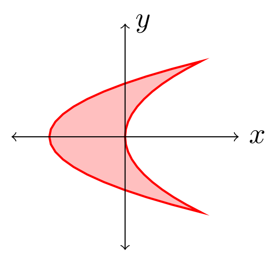
due Friday, October 4, 2024

1. Find the circulation density and the flux density of each of the following vector fields:
   (a) $\mathbf{F} = e^{x^2}\,\mathbf{i} - e^{y^2}\,\mathbf{j}$.
   (b) $\mathbf{F} = \frac{\mathbf{i}+\mathbf{j}}{x^2+y^2}$.
   (c) $\mathbf{F} = (x^3+y^3)\,\mathbf{i} - (x^2y+xy^2)\,\mathbf{j}$.
2. For each part of this problem, come up with an example of a 2-dimensional vector field with the required properties.
   (a) A vector field which is not a constant, but for which both the circulation density and the flux density are 0. (Every constant vector field also has this property; do you see why? But there are also examples that are not constant.)
   (b) A vector field for which the circulation density and flux density are both a positive constant at every point.
   (c) A conservative vector field for which the flux density at (x, y) is equal to $x + y$.
3. Use Green's theorem to compute the counterclockwise circulation of $\mathbf{F} = \frac{1}{x^2+1}\,\mathbf{i} + \frac{2}{3}x^3y^2\,\mathbf{j}$ around the boundary of the square with vertices at $(\pm 1, \pm 1)$.
4. Let R be the region shown below:



This region is bounded by two curves with the parameterizations

$$\mathbf{r}_1(t) = (t^2, t) \qquad t \in [-1, 1]$$
$$\mathbf{r}_2(t) = (2t^2 - 1, -t) \qquad t \in [-1, 1]$$

Use Green's theorem to find the outward flux of $\mathbf{F} = xy\,\mathbf{i} - y\,\mathbf{j}$ across the boundary of R.