Calculus IV Homework 6

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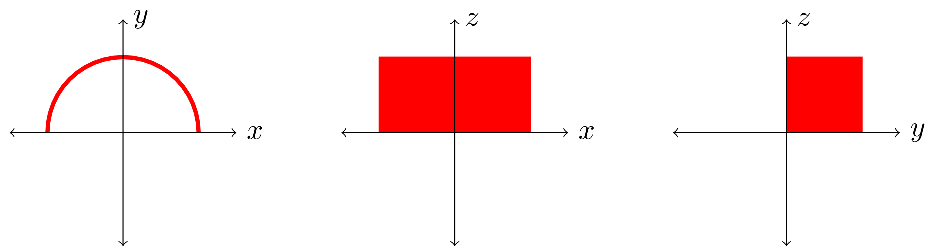
1. For each of the following shapes, draw their projections onto each of the xy-, xz-, and yz-planes, and specify which of these projections are one-to-one.

(example) The half-cylinder given by the equation $x^2 + y^2 = 1$ with $y \geq 0$ and $0 \leq z \leq 1$.

The projections are drawn below: only the projection on the xz-plane is one-to-one.



(a) The portion of the cone $x^2 + y^2 = z^2$ satisfying $0 \leq z \leq 1$.

(b) The portion of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant: where x, y, z are all nonnegative.

(c) The portion of the surface $z = \sin y$ satisfying $-1 \leq x \leq 1$ and $-\pi \leq y \leq \pi$.

2. Evaluate the scalar surface integral

$$\iint_S (y + z)\, \mathrm{d}A$$

where S is the surface parameterized by $\mathbf{r}(u, v) = (2uv, u^2 - v^2, u^2 + v^2)$, $(u, v) \in [0, 5] \times [0, 3]$.

3. Let S be the surface in the shape of the parabolic bowl $z = x^2 + y^2$ with $z \leq 1$. Find the flux of $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j}$ across S in the *downward* direction.

4. Let S be the unit sphere centered at the point $(0, 0, 1)$: the surface with equation $x^2 + y^2 + (z - 1)^2 = 1$. Let's say that we are interested in the *outward* flux of $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across the surface of S. Though S does not have a one-to-one projection onto the xy-plane, we can still use the implicit surface method—provided we split the integral up into two integrals.

(a) Set up an integral for the flux of $\mathbf{F}$ across the top half of S: the portion of S with $z \geq 1$.

Do not evaluate, but do reduce it to an iterated double integral with respect to x and y, with no other variables, and simplify the integrand as much as possible.

Be sure to use the appropriate orientation of S.

(b) Do the same thing for the bottom half of S: the portion of S with $z \leq 1$.