Graph Theory Homework 3

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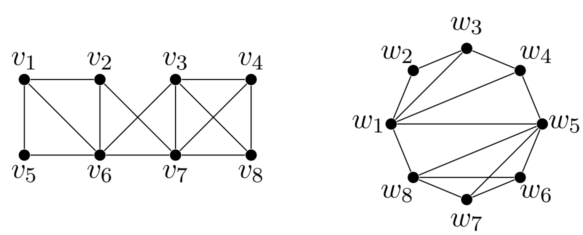
1 Short answer

1. Determine which of the sequences below are graphic sequences. For the ones that are graphic, find a graph with that degree sequence.
   (a) $7, 3, 3, 3, 3, 3, 3, 3$.
   (b) $6, 5, 4, 4, 3, 1, 1$.
   (c) $5, 5, 3, 3, 2, 2, 2$.
2. Find *two* different isomorphisms between the two graphs below:



3. Suppose that an n-vertex tree has 4 vertices of degree 3 and $n - 4$ vertices of degree 1.
   (a) Determine the value of n, and give an example of such a tree.
   (b) Find a second example not isomorphic to the first; explain why they are not isomorphic.

2 Proof

4. In this problem, G and H are two graphs that share some, but not all, of their vertices.

   We write:

   - $G \cap H$ for the graph whose vertices are $V(G) \cap V(H)$ and whose edges are $E(G) \cap E(H)$: all the vertices and all the edges that G and H have in common.
   - $G \cup H$ for the graph whose vertices are $V(G) \cup V(H)$ and whose edges are $E(G) \cup E(H)$: all the vertices and all the edges present in either G or H.

Suppose that G, H, and $G \cap H$ are trees. Prove that $G \cup H$ is a tree.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 4.

5. Prove the following by induction on n. For all $n \geq 5$, there exists a graph with n vertices and $2n - 4$ edges that has minimum degree 2 and maximum degree 4.

   You have already written a rough draft of the solution; now, write a final draft.