Graph Theory Homework 4

Mikhail Lavrov

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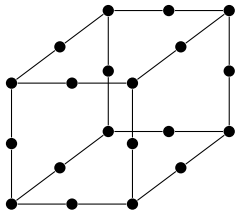
1 Short answer

1. (a) Find the tree with vertex set $\{1, 2, \ldots, 8\}$ and Prüfer code 121336.

   (b) In total, how many different trees with vertex set $\{1, 2, \ldots, 8\}$ have a Prüfer code of $1x1336$ for some x?

2. The graph below is called a *subdivision* of the cube graph; it is obtained by adding a new vertex in the middle of every edge of the cube graph Q_3.



   In this graph, find a matching M and a vertex cover U with $|M| = |U|$.

2 Proof

3. What is the largest possible size of a matching in a tree T with 60 vertices, 40 of which are leaves? You should prove both parts of the answer: if you say "the largest possible size is m", then you should give an example of a 60-vertex, 40-leaf tree with a matching of size m, *and* prove that there is no 60-vertex, 40-leaf tree with a matching of size $m + 1$ or more.

   Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 5.

4. In this problem, G and H are two graphs that share some, but not all, of their vertices.

   We write:

   - $G \cap H$ for the graph whose vertices are $V(G) \cap V(H)$ and whose edges are $E(G) \cap E(H)$: all the vertices and all the edges that G and H have in common.
   - $G \cup H$ for the graph whose vertices are $V(G) \cup V(H)$ and whose edges are $E(G) \cup E(H)$: all the vertices and all the edges present in either G or H.

   Suppose that G, H, and $G \cap H$ are trees. Prove that $G \cup H$ is a tree.

   You have already written a rough draft of the solution; now, write a final draft.