

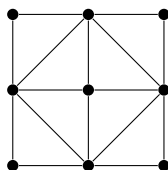
Graph Theory Homework 6

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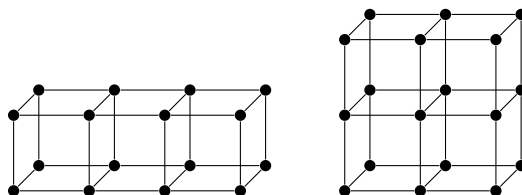
due Friday, October 25, 2024

1 Short answer

1. Find a Hamiltonian path in the graph below, and explain why it does not have a Hamiltonian cycle.



2. For each graph below, determine whether it is planar:



2 Proof

3. Prove that there is no 4-regular planar graph in which all faces have length 4.

Write a rough draft of the solution. I will give you feedback, and you will write a final draft of your proof as part of Homework 5.

4. In a directed graph, it is possible to start at a vertex, follow some of the edges, and then end up unable to return to where you started. In other words, it is possible that a directed graph contains an $x - y$ walk but not a $y - x$ walk for some vertices x and y .

Let's call a vertex x **safe** if it's impossible to leave it and get lost: for every vertex y such that there is an $x - y$ walk, there is also a $y - x$ walk.

Prove that every directed graph has at least one safe vertex.

You have already written a rough draft of the solution; now, write a final draft.