Graph Theory Homework 8

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due Friday, November 22, 2024

1 Course evaluations

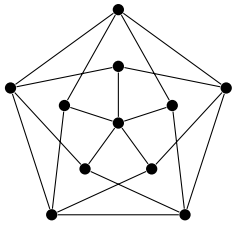
Course evaluations for this class (and other classes) should now be accessible on D2L. The deadline for filling out course evaluations is December 2nd: after classes are over, but before our final exam on December 5th.

These are very useful to me in deciding how to teach this class in the future. (Comments are especially helpful, but even if you don't leave those, numerical ratings are helpful, and they're more reliable if more of you give them.) The evaluations are anonymous, and are not released until after final grades are due, so there are no drawbacks to being honest.

As an incentive, if at least 50% of the class fills out a course evaluation, I will wear a funny hat for the final exam.

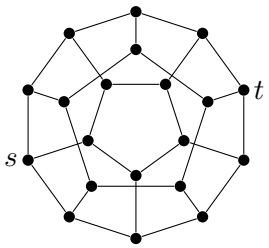
2 Short answer

1. Give an example of:
   (a) A graph with 8 vertices, 6 of which are cut vertices.
   (b) A graph with 9 vertices, exactly one cut vertex, and minimum degree 4.
2. The Grötzsch graph, shown below, is 3-connected.



   Demonstrate this using ear decompositions.

3. Let G be the dodecahedron graph, shown below; let s and t be two opposite vertices of the dodecahedron.



Determine $\kappa(s,t)$; explain why the value cannot be larger or smaller.

4. Answer briefly (one sentence should be enough for each part). Feel free to cite theorems from class if they are relevant.

   (a) In a tree, which vertices are cut vertices?

   (b) Why is every graph with at most one cycle 3-colorable?

   (c) What is the clique number $\omega(K_{m,n})$ of the complete bipartite graph $K_{m,n}$?

   (d) Why does every n-vertex planar graph have an independent set of size at least $n/4$?

3 Proof

5. Determine the minimum number of edges in an $3n$-vertex graph with an independence number of $2n$.

   You have already written a rough draft of the solution; now, write a final draft.