Probability Theory Homework 2

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1. Consider a random experiment in which three fair coins are flipped. Let A be the event "All three coins have the same result."

   (a) List the eight elements of the sample space, and the elements of A.

   (b) Give an example of an event B such that $\Pr[B] = \frac{1}{2}$, and A and B are disjoint.

   (c) Give an example of an event C such that $\Pr[C] = \frac{1}{2}$, and A and C are independent.

2. An insurance company offers home insurance policies and car insurance policies. Of their clients, 25% just have home insurance, 40% just have auto insurance, and 35% have both.

   Based on past data, each client with home insurance has a 60% chance of renewing that policy next year, and each client with car insurance has a 50% chance of renewing that policy next year. (For clients with both types of insurance, the decisions appear to be independent.)

   Assuming these trends continue:

   (a) What is the probability that a client with both types of insurance renews at least one policy?

   (b) What is the probability that a randomly chosen client renews at least one policy?

3. In the board game Monopoly, if you end up in Jail, then on your turn you roll two fair 6-sided dice; if you roll doubles (the same number on both dice), you get out, and otherwise you stay in Jail and skip your turn. Usually you can also get out by paying a fine, but let's say that we are playing with house rules that prevent that.

   Suppose that you're playing Monopoly and it takes you *seven or more* turns to get out of Jail. Given this information, what is the probability that it takes you *exactly eight* turns?

4. Alice has two coins: a fair coin, and a biased coin which lands on Heads with a probability p that Alice knows (but you don't).

   You choose one of Alice's coins uniformly at random. You flip it once, and it lands on Heads. Alice does some calculations and tells you that, given this result, the probability that you flipped the biased coin is $\frac{3}{5}$.

   Suppose you flip the coin again, and it lands on Heads again. Given the results of both coinflips, what is the probability that you flipped the biased coin?

5. You have three bags of marbles, as shown below: the first has 3 blue marbles and 1 red marble, the second has 3 blue marbles and 2 red marbles, and the third has 1 blue marble and 2 red marbles.
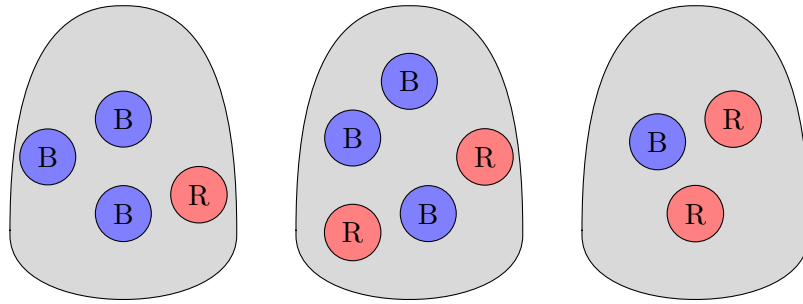
(a) If you pick a bag uniformly at random and draw a marble from it, what is the probability that the marble will be red?

(b) If you pick a bag uniformly at random and draw a marble from it, and the marble turns out to be red, what is the probability that the bag you picked was the third bag?