

# Probability Theory Homework 3

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1. A bag of marbles contains 3 red, 4 blue, and 5 green marbles.

Suppose I tell you that I took 3 marbles out of the bag (without replacement), and they were all the same color. What is the probability that all 3 marbles were blue?

2. You roll a fair six-sided die three times. What is the probability that you roll three different numbers in ascending order? (*For example, 1, 3, 6 counts, because  $1 < 3 < 6$ , but 3, 6, 1 does not count, because 6 is not less than 1, and 2, 2, 5 does not count, because 2 is not less than 2.*)
3. You draw 4 cards from a standard 52-card deck. (*There are 4 suits in the deck; there are 13 cards of each suit.*)

What is the probability that you draw one card of each suit if:

- (a) You draw a hand of 4 cards, so you are holding them all at once?
  - (b) You draw 4 cards one at a time, putting each card back and shuffling before you draw the next?
4. A biased coin has a  $\frac{2}{3}$  probability of landing heads and a  $\frac{1}{3}$  probability of landing tails. You toss the coin 9 times. Which of the two options below is more likely to happen, and why?
    - The coin lands heads only 3 times, and lands tails 6 times.
    - The coin lands heads all 9 times.
  5. Consider the following random experiment: you roll two dice whose faces have the labels  $\square, \square, \odot, \odot, \oplus, \oplus$ . That is, when you roll one die, you get a result of 0, 1, or 2 with equal probability.

- (a) Write down the “big” sample space for this random experiment: the one which gives us the results of both rolls as the outcome. (There are two plausible ways to do this; pick the one where we sample uniformly from this sample space.)
- (b) Let  $\mathbf{X}$  be the random variable which gives the larger of the two numbers rolled. For example, if the two rolls are  $\square$  and  $\odot$ , then  $\mathbf{X} = 1$ .

Describe  $\mathbf{X}$  as a function from the sample space to its range  $R_{\mathbf{X}} = \{0, 1, 2\}$ . (I’m happy with any way of conveying the information, as long as you tell me which inputs map to which outputs.)

- (c) Write down the probability mass function  $P_{\mathbf{X}}(k)$ .