

Probability Theory Homework 5

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1. You take a 20-question multiple choice exam on which every correct answer is worth 1 point, and every incorrect answer is worth $-\frac{1}{4}$ points to discourage guessing. Each question has five options: (A) through (E). On each question, you are able to eliminate one of the options as certainly wrong (leaving four), and then guess randomly between the other four options.
 - (a) Let \mathbf{X} be the number of correct answers you give. What distribution does \mathbf{X} have (one of the named distributions we covered), and what are its parameters?
 - (b) Express the number of points you receive as a linear transformation $a\mathbf{X} + b$ of \mathbf{X} , the random variable from part (a).
 - (c) Find the probability that you get exactly 12.5 points.
2. Suppose that someone offers you a sequence of bets on the outcomes of rolling a die. Here is how the betting works.

A prize pool starts out at \$1000. The die is rolled, and if it comes up 6, the prize pool is doubled, and the entire process is repeated. The game only ends once the number that comes up is not 6, at which point you win the amount in the prize pool.

(For example, if the rolls are 6, 6, 6, 5, the prize pool ends up doubling three times, and you win \$8000.)

 - (a) Express the amount of money you win as a transformation of a $\text{Geometric}(p = \frac{5}{6})$ random variable.
 - (b) Find the expected amount of money you win.
3.
 - (a) Find the conditional PMF of $(\mathbf{H} \mid \mathbf{H} \geq 3)$, where $\mathbf{H} \sim \text{Hyper}(n = 5, b = 6, r = 4)$.
 - (b) Find the expected value $\mathbb{E}[\mathbf{H} \mid \mathbf{H} \geq 3]$.
4. Suppose that the number of times that you go to the doctor each year has a Poisson distribution with an average of 2. Each visit to the doctor costs \$500.

Your insurance will cover *all* of your medical fees if you end up going to the doctor 5 or more times. If you end up going to the doctor 4 or fewer times, you have to pay your medical fees yourself.

 - (a) Find the expected amount of money you have to pay for medical fees in a year.
 - (b) Find the conditional expected amount of money you have to pay for medical fees, given that you ended up having to pay the fees yourself.
5. Find the variance of a fair six-sided die whose sides are labeled 1, 2, 2, 3, 3, 3.