

Probability Theory Homework 6

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due Friday, October 25, 2024

1. At the Skittles factory, a bag of Skittles is filled by a mechanical scoop. The scoop picks up 9, 10, 11, or 12 Skittles (with equal probability of each number) and pours them into the bag; this is repeated a total of 5 times, resulting in a bag which contains between 45 and 60 Skittles.
 - (a) Find $\text{Var}[\mathbf{S}]$, where \mathbf{S} is the number of Skittles scooped up by the scoop.
 - (b) Find $\text{Var}[\mathbf{B}]$, where \mathbf{B} is the total number of Skittles in the bag.
2. You are lost in the woods. You don't have a sense of direction, so you wander randomly according to the following rules:
 - From the Center of the woods, you are equally likely to go North, East, South, or West.
 - If you go North, you reach a dead end, from which you are certain to return to the Center.
 - If you go South, you hit a tricky dead end where it's easy to get turned around. You have a $\frac{1}{2}$ chance of returning to the Center, and $\frac{1}{2}$ of ending up at the same location South of the center.
 - If you go West, you hit a *very* tricky dead end; you have a $\frac{1}{3}$ chance of returning to the Center and a $\frac{2}{3}$ chance of ending up at the same location West at the center.
 - If you go East, you find the exit and leave.

If each step in this random walk (whether from one location to another, or from a location back to itself) takes you 1 minute, what is the expected number of minutes it takes you to leave the woods?

3. You have two dice: a fair die (which lands on 6 with probability $\frac{1}{6}$) and a loaded die (which lands on 6 with probability $\frac{1}{2}$). You pick one of the dice at random and roll it until a 6 is rolled.

What is the expected number of rolls this takes you?

4. Renting ice skates at a skating rink costs \$5.00. Rather than do this, Alice decides to buy cheap skates for \$25.00. However, the cheap skates have a $\frac{1}{10}$ chance of breaking down irreparably after every time Alice uses them.

Once they break, Alice computes \mathbf{Y} , the amount of money she saved compared to renting skates an equal number of times. (Note that \mathbf{Y} might be negative: if the skates break the first time, then $\mathbf{Y} = -20$, because it would have been 20 dollars cheaper to rent.)

- (a) What is the distribution of \mathbf{X} : the number of times Alice uses the cheap skates?
- (b) What is \mathbf{Y} , as a function of \mathbf{X} ?
- (c) Find $\mathbb{E}[\mathbf{Y}]$ and $\text{Var}[\mathbf{Y}]$.